Quantum transport in the presence of oscillating spin-orbit interaction

Viktor Szaszkó-Bogár

Department of Theoretical Physics
University of Szeged

SEMINAR

04.12.2014

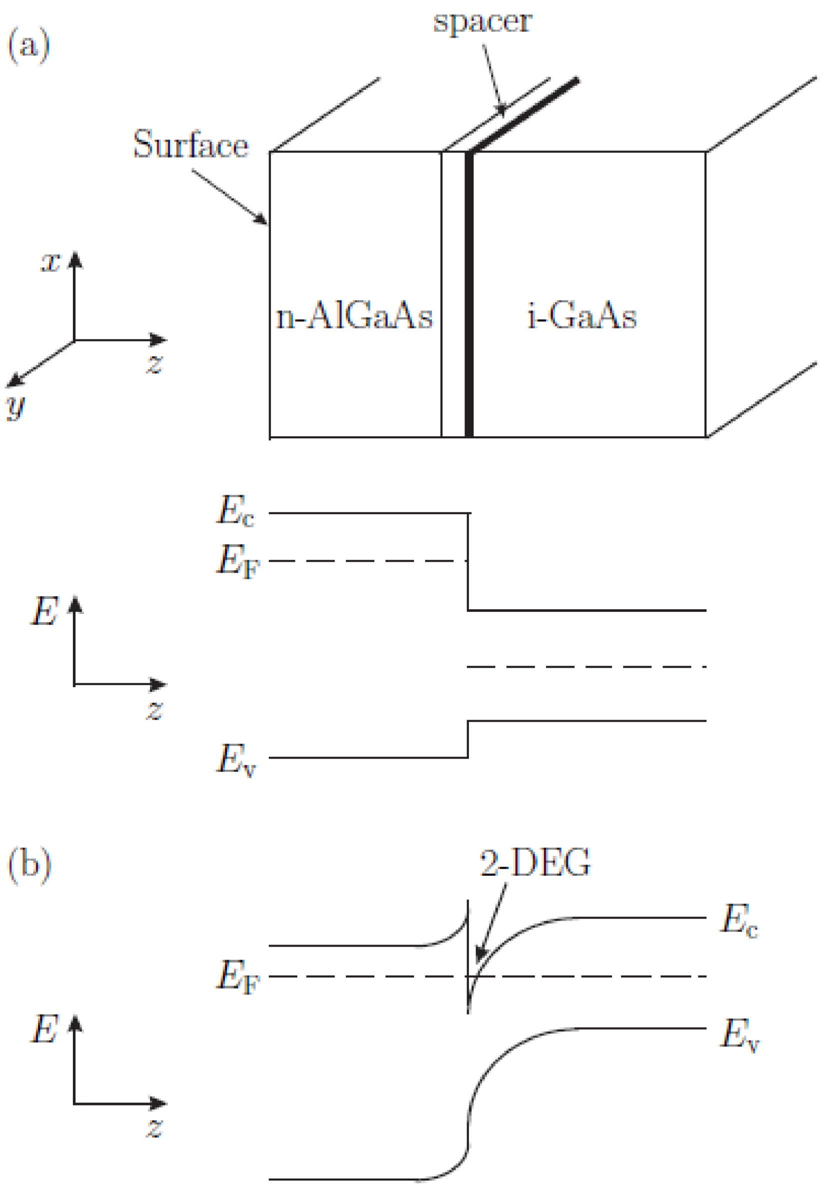
Outline

- Introduction
  - Semiconductor heterostructures
  - Ohmic and ballistic regime
  - Contact resistance
  - Derivation of conductance: Landauer-Büttiker approach
  - Spin-orbit interaction (SOI)
  - Rashba- and Dresselhaus spin-orbit fields
- Transport in the presence of oscillating SOI
  - Loop geometries as nanodevices
  - Time-dependent Hamiltonian and the Floquet-theory
  - High harmonics
  - Floquet quasi-energies
  - Oscillating spin direction
  - Oscillating density waves
  - Results

Part I

Introduction

Semiconductor heterostructure



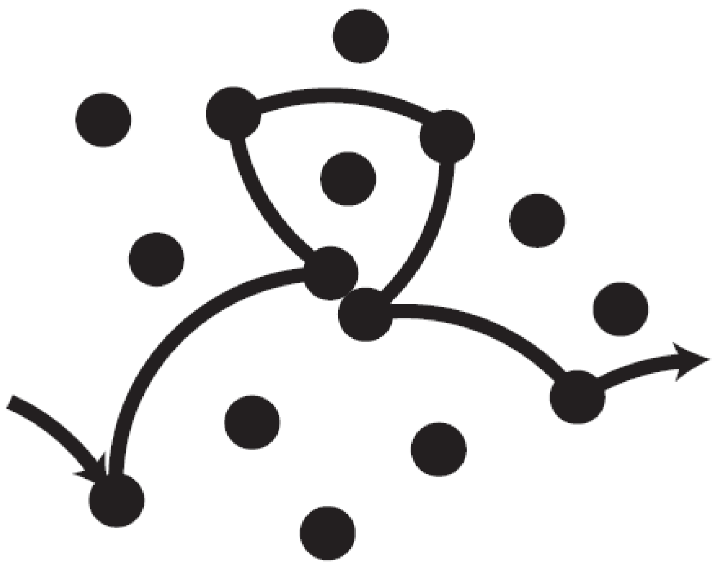
The Fermi energy in the widegap AlGaAs layer is higher than in the narrowgap GaAs layer.
Consequently electrons spill over from the n-AlGaAs leaving behind positively charged donors.

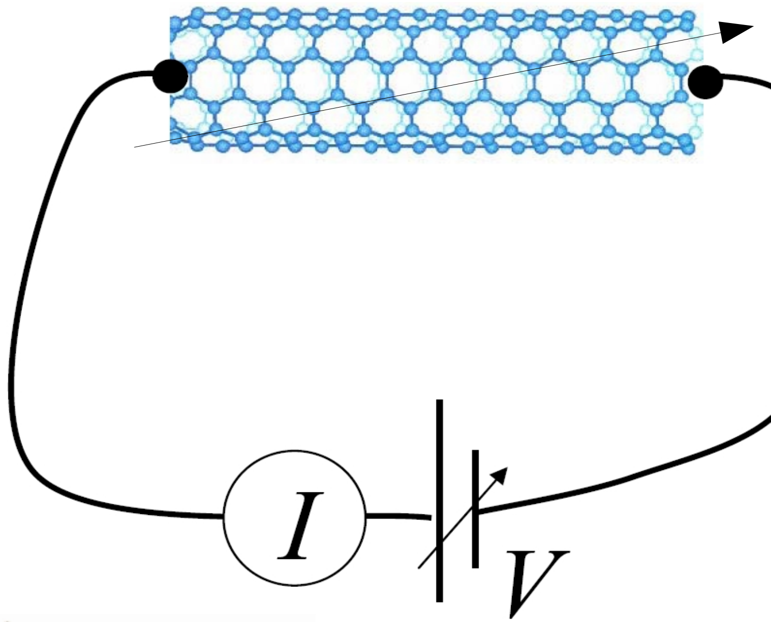
This space charge gives rise to an electrostatic potential.

At equilibrium the Fermi energy is constant everywhere.
The electron density is sharply peaked near the AlGaAs-GaAs interface.

Thin conducting layer:
two-dimensional electron gas (2-DEG)

Ohmic and ballistic regime





Classical (incoherent) transport	Ballistic (coherent) transport
Drude model	No scaterring effects
Ohm's law	Quantized conductance
Scattering mechanisms	
$L_\phi, L_m \ll L$	$L_\phi, L_m \gg L$

L_ϕ Phase relaxation length; L_m Momentum relaxation length; L Sample length

Short historical review

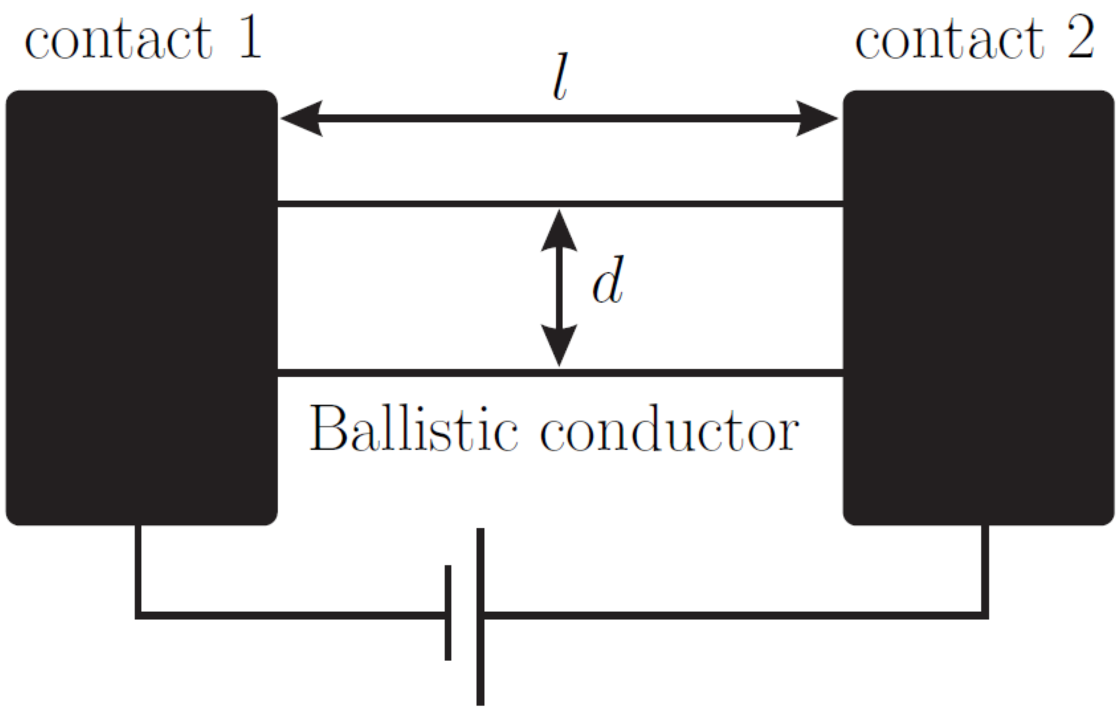
Rolf William Landauer
1927-1999

Markus Büttiker
1950-2013

R. Landauer,
"Spatial variation of currents and fields due to localized scatterers in metallic conduction,"
IBM J. Res. Develop.1, 223 (1957)

M. Büttiker, Y. Imry, R. Landauer, and S. Pinhas,
"Generalized many-channel conductance formula with application to small rings,"
Phys. Rev.B 31, 6207 (1985)

Contact resistance I.



A conductor is placed between two contacts across which an external bias is applied.

Macroscopic regime:

$$G = \sigma \frac{d}{l},$$

Ohm's law

Conductivity: σ

Mesoscopic regime:

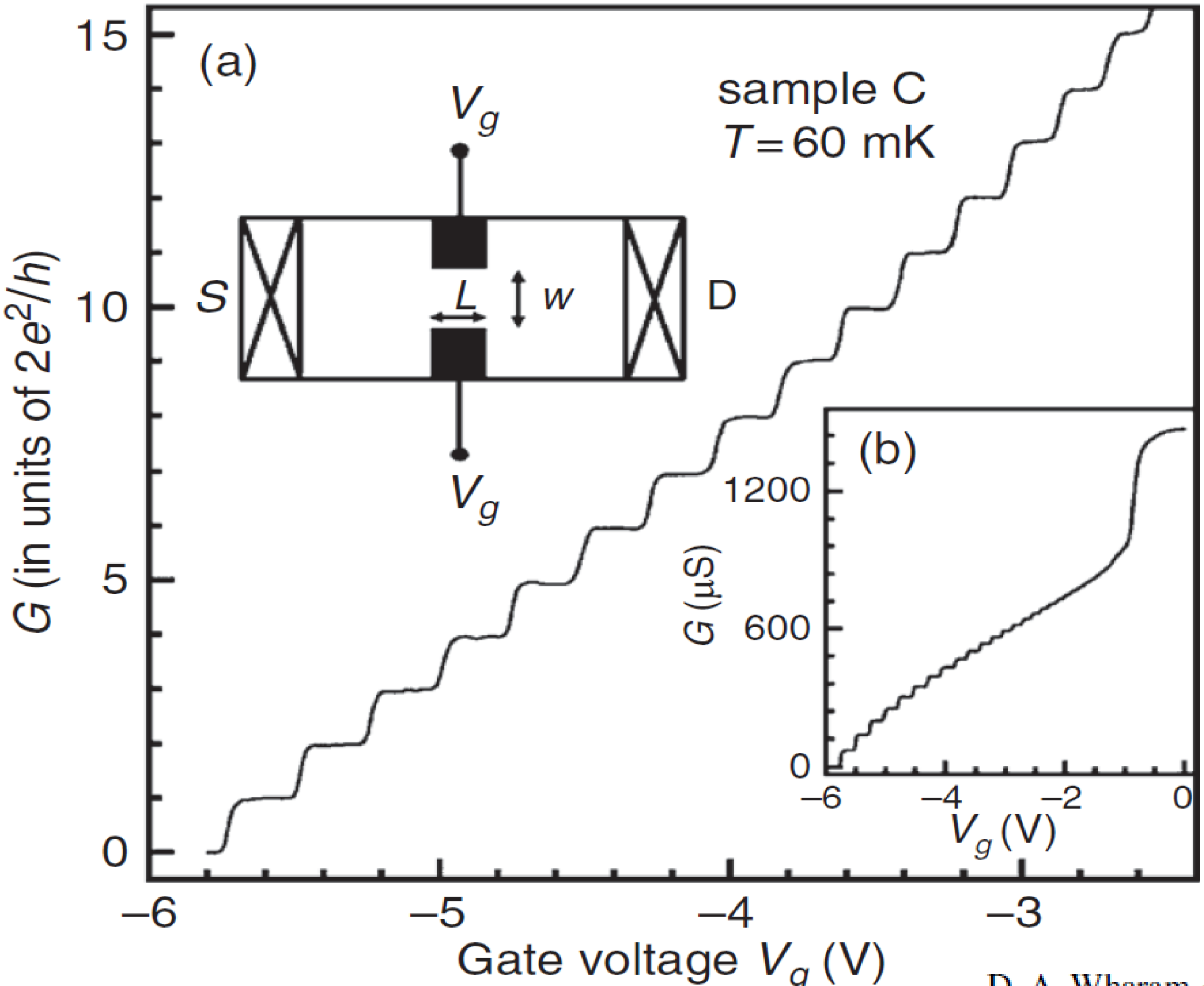
We would expect the resistance to become zero.

$$\lim_{d,l \to 0} G^{-1} = 0.$$ **WRONG!**

$$\lim_{d,l \to 0} G^{-1} = R_C.$$ **CORRECT!**

Contact resistance II.

Experimental observations



Quantized conductance of a ballistic conductor.

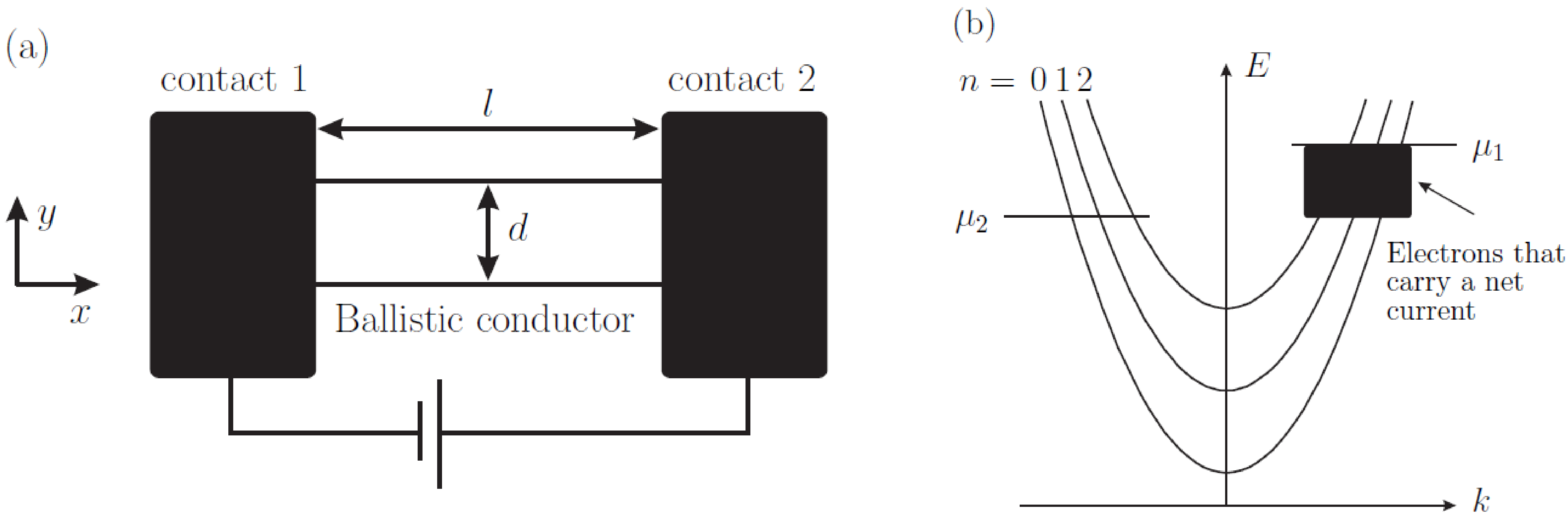
$$R_C = \frac{h}{2e^2 M} \approx \frac{12.9\,\mathrm{k}\Omega}{M}$$

Number of transverse modes: M

B. J. van Wees *et al.*, Phys. Rev. Lett. **60**, 848 (1988).

D. A. Wharam *et al.*, J. Phys. C: Solid State Physics **21**, L209 (1988)

Landauer formula I.



Reflectionless contacts:
The electrons can enter them from the conductor without suffering reflections.

Assuming that *M* modes carry the current the contact resistance (which is the resistance of a ballistic waveguide) is given by

$$G_C^{-1} = \frac{h}{2e^2}\frac{1}{M}$$

Landauer formula II.

The current that enters the conductor is: $$I_{in} = \frac{2e}{h} M \left(\mu_1 - \mu_2\right)$$

The current which flows out of the conductor is: $$I_{out} = T \frac{2e}{h} M \left(\mu_1 - \mu_2\right)$$

T transmission probability

The external bias: $$U = \frac{\mu_1 - \mu_2}{e}$$

Finally we find the form of conductance

$$G = \frac{2e^2}{h} MT$$

Landauer formula (1957)

Expansion of the Dirac equation

In this case of an electron which is moving in an external field, the Dirac equation may be written as

$$i\hbar\frac{\partial\Psi}{\partial t} = \left\{c\hat{\boldsymbol{\alpha}}\left(\hat{\mathbf{P}} - \frac{e}{c}\mathbf{A}\right) + mc^2\hat{\beta} + e\Phi\right\}\Psi,$$

where

$$\hat{\alpha}_k = \begin{pmatrix} \hat{0} & \hat{\sigma}_k \\ \hat{\sigma}_k & \hat{0} \end{pmatrix}, \quad k = 1, 2, 3, \quad \hat{\beta} = \begin{pmatrix} \hat{I} & \hat{0} \\ \hat{0} & -\hat{I} \end{pmatrix},$$

Dirac bispinor: $\Psi(\mathbf{r}) = \begin{pmatrix} \varphi \\ \chi \end{pmatrix}.$

In the non-relativistic approximation and taking an expansion of the Dirac equation up to second order $1/c^2$:

$$H = \frac{\mathbf{P}^2}{2m} + e\Phi - \frac{\mathbf{P}^4}{8m^3c^2} \boxed{- \frac{e\hbar}{4m^2c^2}\boldsymbol{\sigma}\cdot(\mathbf{E}\times\mathbf{P})} - \frac{e\hbar^2}{8m^2c^2}\nabla\mathbf{E}.$$

SPIN-ORBIT INTERACTION TERM

Rashba and Dresselhaus spin-orbit fields I.

If the electric field has central symmetry, we can write

$$\mathbf{E} = -\hat{\mathbf{r}}\frac{\mathrm{d}\Phi}{\mathrm{d}r},$$

and the spin-orbit interaction operator can be expressed in the following form

$$H_{SO} = \frac{\hbar^2}{2m^2c^2r}\frac{\mathrm{d}V}{\mathrm{d}r}\mathbf{L}\cdot\mathbf{S},$$

where the angular momentum operator is **L**, and the electron spin operator is **S**.

Time reversal symmetry:

$$t \to -t$$
$$\sigma \to -\sigma$$
$$L \to -L$$

➡ $H_{SO} \sim L \cdot S \to H_{SO}$

+

SOI preserves time reversal symmetry!

Space inversion symmetry: $\varepsilon_\uparrow(k) = \varepsilon_\downarrow(k)$ **Degeneracy!**

Space inversion asymmetry: $\varepsilon_\uparrow(k) \neq \varepsilon_\downarrow(k)$

Rashba and Dresselhaus spin-orbit fields II.

Time reversal symmetry ONLY: $\varepsilon_{\uparrow}(k) = \varepsilon_{\downarrow}(-k)$ and $\varepsilon_{\uparrow}(k) \neq \varepsilon_{\downarrow}(k)$

What happens when inversion symmetry is broken?

↓

Emergence of spin-orbit fields

Bulk inversion asymmetry (BIA) ➞ **Dresselhaus SO field**

(zinc blende semiconductors [GaAs, InAs])

Structure inversion asymmetry (SIA) ➞ **Rashba SO field**

(relevant in 2-DEG)

Rashba Hamiltonian: $H_{RSOI} = \alpha_R \left(\sigma_x k_y - \sigma_y k_x\right)$

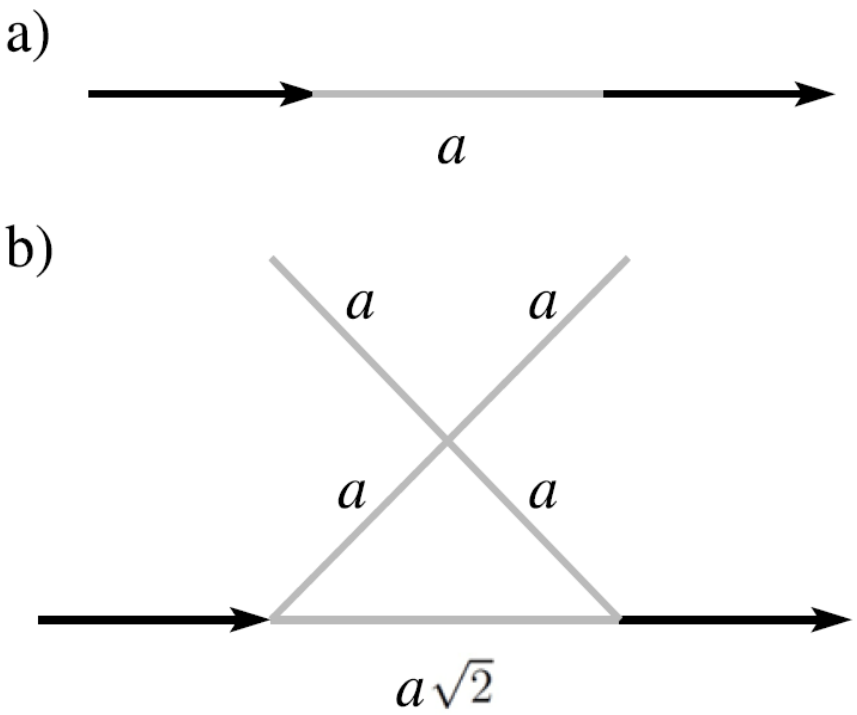
α_R Rashba-parameter which can be tuneable by an external gate voltage.

J. Nitta, T. Akazaki, H. Takayanagi, and T. Enoki, Phys. Rev. Lett. 78, 1335 (1997).

Part II

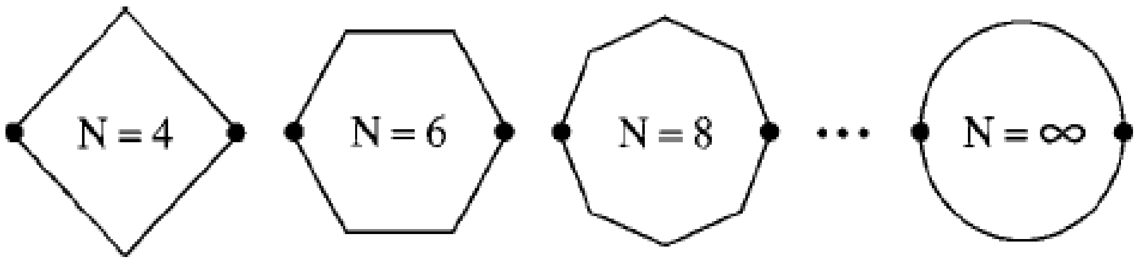
Transport in the presence of oscillating SOI

Model of loop geometries



Gray lines correspond to quantum wires with oscillating SOI. We assume no spin–orbit interaction in the input/output leads that are indicated by the black arrows.

D. Bercioux, D. Frustaglia, and M. Governale, Phys. Rev. B 72, 113310 (2005).



Series of regular-polygon conductors of constant perimeter. Vertices are connected by single-channel ballistic quantum wires with SO coupling. In the limit of infinite number of vertices the series converges to a single-channel circular conductor. The full dots represent the point where input and output leads are attached.

Time-dependent Hamiltonian I. Floquet-theory

Theorem

The basic solutions to the time-dependent Schrödinger equation with time-periodic Hamiltonian $H(t+T) = H(t)$ can be given in the form

$$|\Psi_\alpha(t)\rangle = e^{-\frac{i}{\hbar}\varepsilon_\alpha(t-t_0)}|\phi_\alpha(t)\rangle,$$

where ε_α are the Floquet exponents and $|\phi_\alpha(t)\rangle$ are the time-periodic Floquet states, which are solutions to the Floquet-type Schrödinger equation

$$\mathcal{H}|\phi_\alpha(t)\rangle = \left(H - i\hbar\frac{\partial}{\partial t}\right)|\phi_\alpha(t)\rangle = \varepsilon_\alpha|\phi_\alpha(t)\rangle.$$

The ε_α and $|\phi_\alpha(t)\rangle$ are called as quasi-eigenenergies and quasi-eigenstates.

Time-dependent Hamiltonian II.

The relevant time-dependent Hamiltonian:

$$\tilde{H}(t) = \hbar\Omega\left[\left(-\mathrm{i}\frac{\partial}{\partial s} + \frac{\omega(t)}{2\Omega}\mathbf{n}(\sigma \times \mathbf{e}_z)\right)^2 - \frac{\omega(t)^2}{4\Omega^2}\right]$$

This Hamiltonian depends on time via the strength of the SOI:

$$\omega(t) = \frac{\alpha(t)}{a}$$

In the following we assume:

$$\omega(t) = \omega_0 + \omega_1 \cos(\tilde{\nu} t).$$

Introducing dimensionless units:

$\tau = \Omega t$ Dimensionless time

$H = \tilde{H}/\hbar\Omega$ Dimensionless Hamiltonian

$\nu = \tilde{\nu}/\Omega$ Dimensionless frequency

Schrödinger equation:

$$\mathrm{i}\frac{\partial}{\partial \tau}|\psi\rangle(\tau) = H(\tau)|\psi\rangle(\tau)$$

High harmonics

Jacobi-Anger identity:

$$e^{iz\sin(\theta)} = \sum_{n=-\infty}^{\infty} J_n(z) e^{in\theta},$$

Where n is an integer and functions J(z) are the Bessel functions of first kind.

Emergence of higher harmonics in time-dependent basis states:

$$|\varphi^{\pm}\rangle(s,\tau) = \mathrm{e}^{-\mathrm{i}\varepsilon^{\pm}(k)\tau} \sum_{n=-\infty}^{+\infty} J_n\left(\frac{k\omega_1}{\Omega\nu}\right) \boxed{\mathrm{e}^{\mp \mathrm{i}n\nu\tau}} |\varphi^{\pm}\rangle(s),$$

An infinite number of additional ‘Floquet channels’ corresponding to frequencies

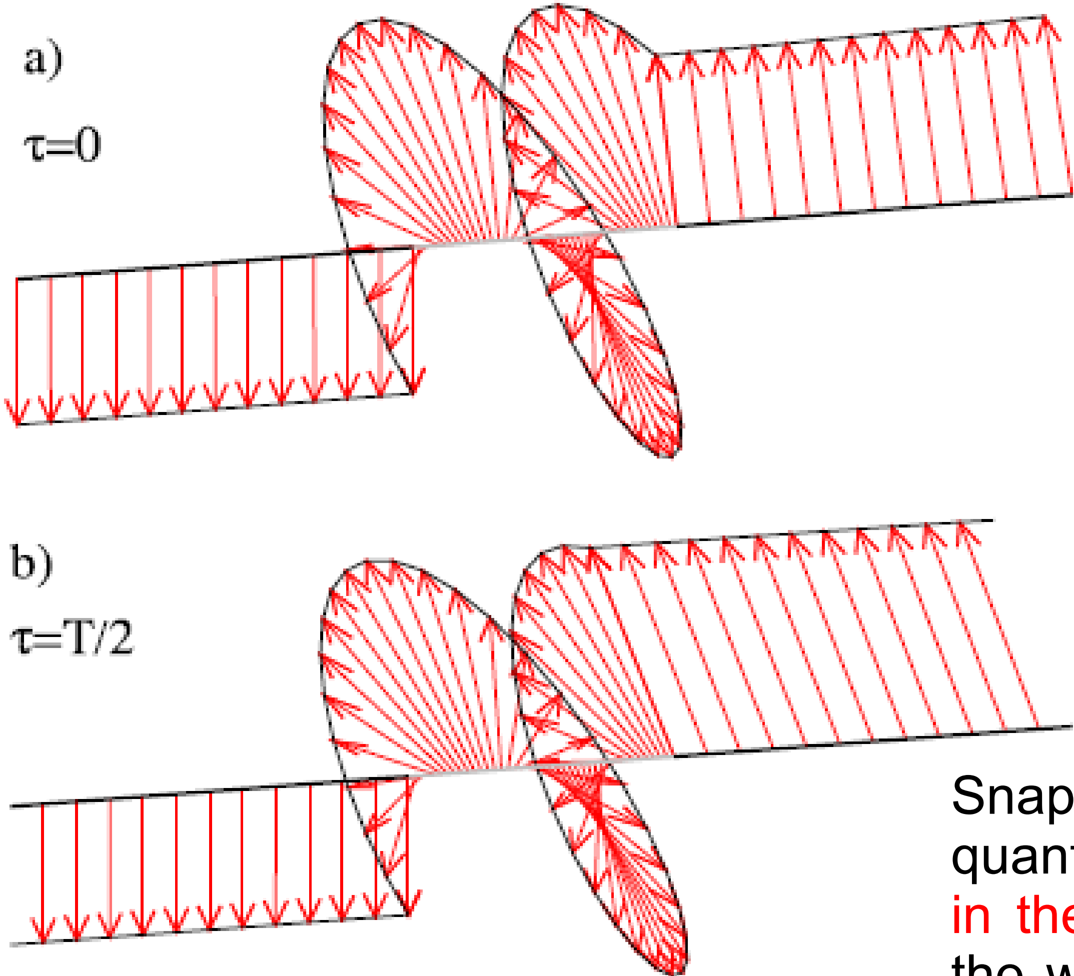
$$\epsilon_n = \epsilon_0 + n\nu$$

open for transmission (with n being an integer).

Results I.

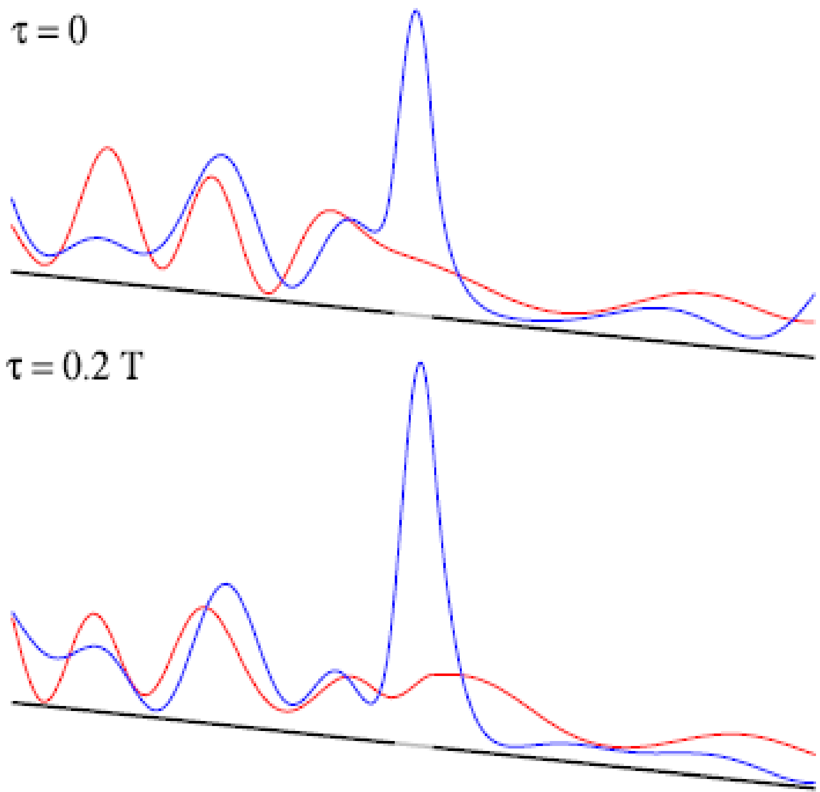
The relevant parameters:

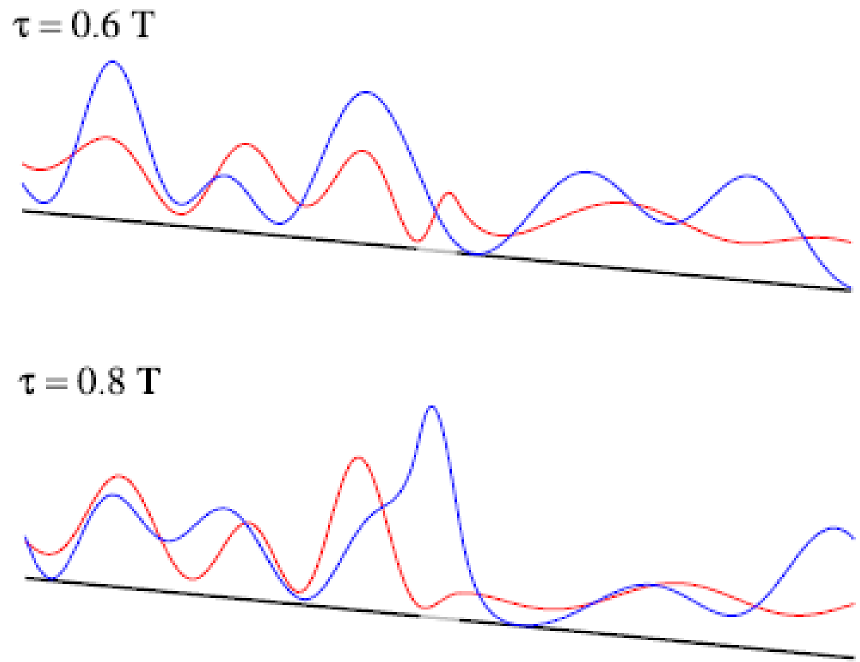
$$\omega_0/\Omega = \omega_1/\Omega = 9.0$$
$$\nu = 1.0$$
$$k_0 a = 1.5$$

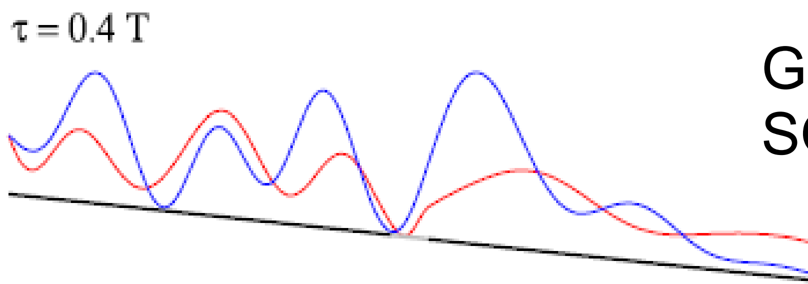


Snapshots of the spin direction along a quantum wire. Oscillating SOI is present in the central region (where the color of the wire is gray). The thin black line that connects the arrowheads is plotted in order to guide the eye.

Results II.







The relevant parameters:

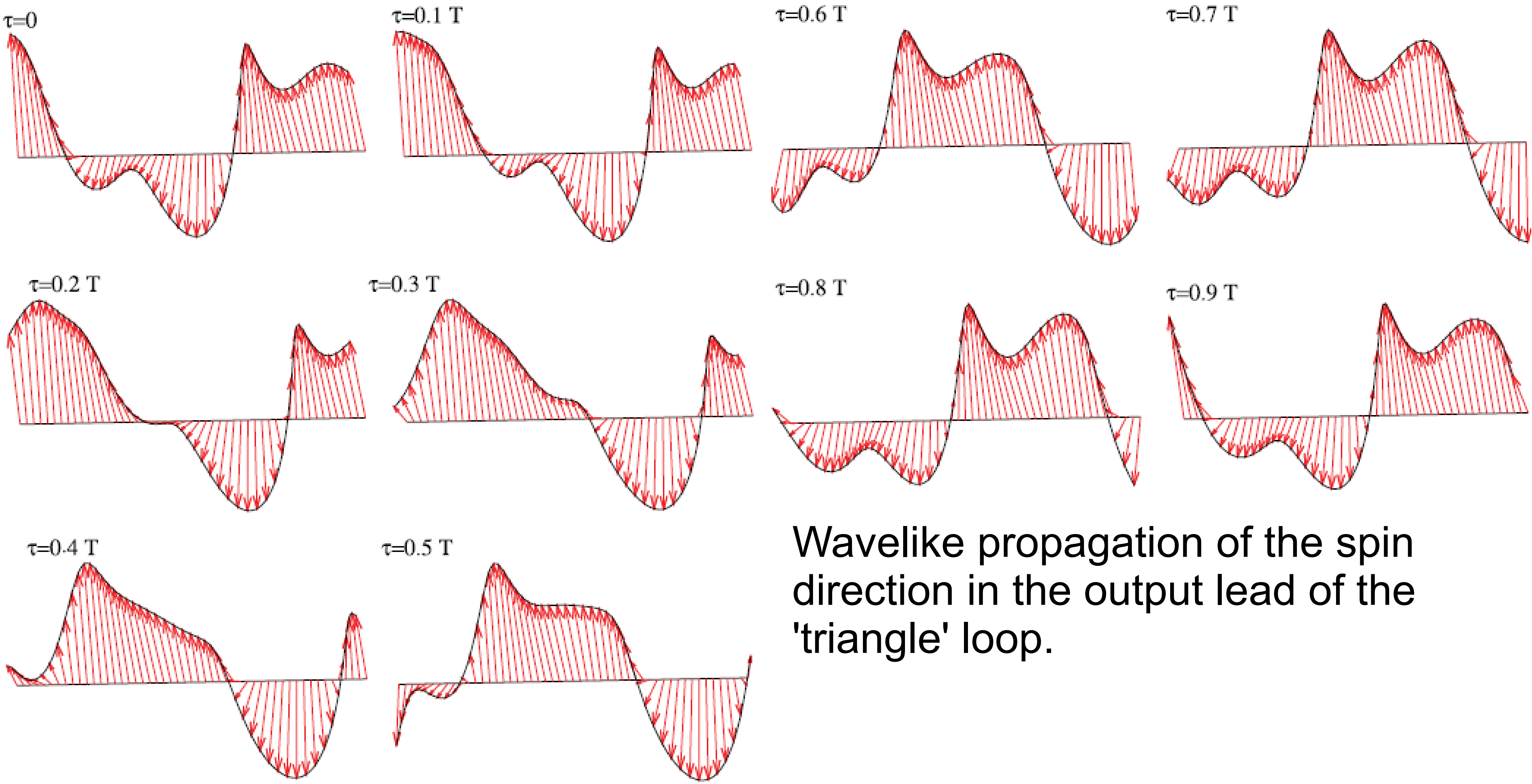
$\omega_0/\Omega = 2.5$ $\qquad$ $\omega_1/\Omega = 2.0$

$\nu = 1.0$ $\qquad$ $k_0 a = 1.0$

Generation of density waves by the oscillating SOI in a quantum wire.

$$\rho(s,\tau) = \langle \Psi | \Psi \rangle (s,\tau)$$

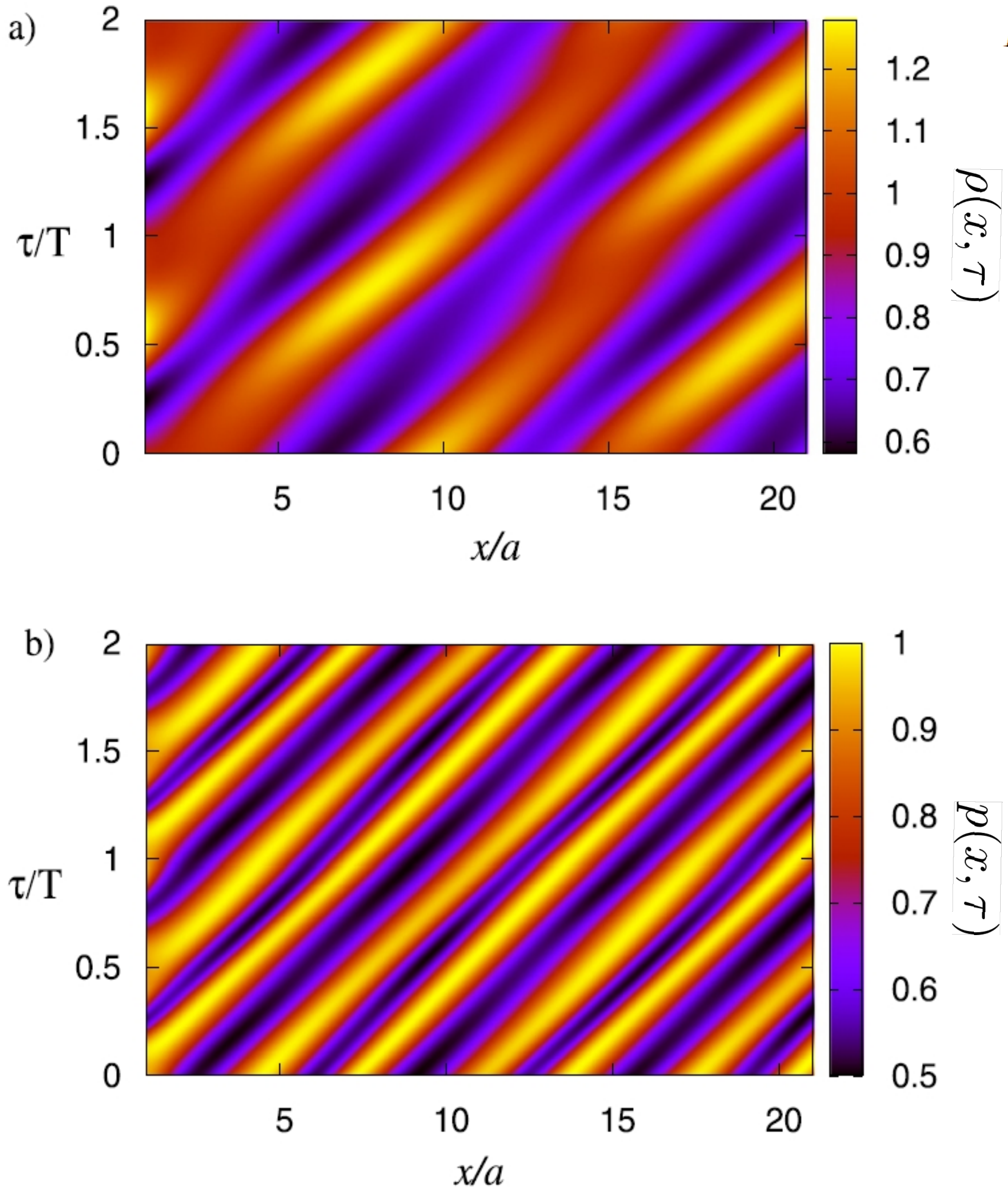
Results III.



Wavelike propagation of the spin direction in the output lead of the 'triangle' loop.

Relevant parameters: $\omega_0/\Omega = 3.0 \quad \omega_1/\Omega = 1.0 \quad k_0 a = 1.5$

Results IV.



$$P(s,\tau)=\frac{1}{2}\left[|\Psi_\uparrow\rangle\langle\Psi_\uparrow|(s,\tau)+|\Psi_\downarrow\rangle\langle\Psi_\downarrow|(s,\tau)\right]$$

$$\rho(s,\tau)=\mathrm{Tr}[P(s,\tau)]$$

$$p(x,\tau)=\frac{1}{[\rho(x,\tau)]^2}\mathrm{Tr}[P^2(x,\tau)]$$

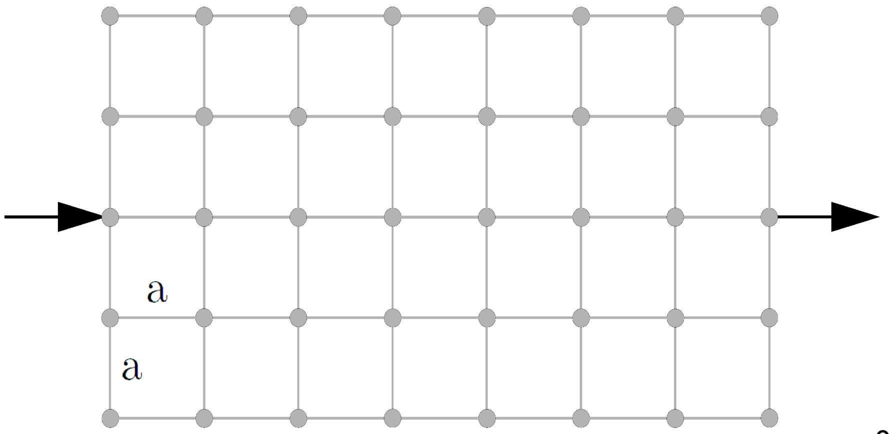
The output corresponding to a completely unpolarized Input for the 'triangle' loop geometry.

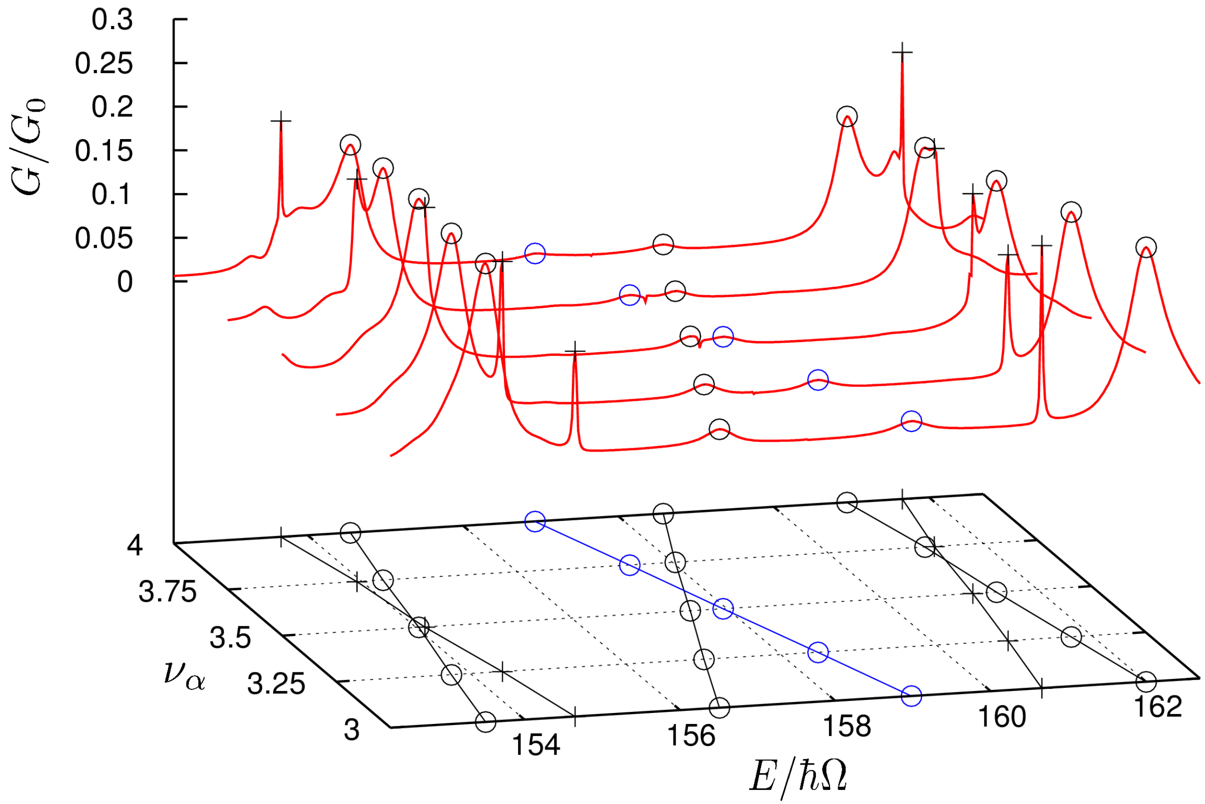
The density $\rho(x,\tau)$ is not normalized.

Relevant parameters:

$$\omega_0/\Omega = 3.0 \qquad \nu = 1.0$$
$$\omega_1/\Omega = 0.3 \qquad k_0 a = 1.0$$

Current work





Summary

- We investigated spin-dependent quantum transport through devices in which the spin–orbit interaction (SOI) is time-dependent, more precisely, it oscillates.
- We have shown the emergence of electron density and spin polarization waves propagating away from their source, i.e. the region with oscillating SOI.
- It was demonstrated that simple geometries can produce spinpolarized wavepackets even for completely unpolarized input.
- Our model suggests a novel source of spin-polarized electrons that can be realized with pure semiconducting materials without the use of external magnetic fields.

References

[1] S. Datta, Electronic transport in mesoscopic systems (Cambridge University Press, Cambridge, 1995).
[2] T. Ihn, Semiconductor Nanostructures (Oxford University Press, New York, 2010).
[3] I. Žutic, J. Fabian, and S. D. Sarma, Rev. Mod. Phys. 76, 323 (2004).
[4] J. Nitta, T. Akazaki, H. Takayanagi, and T. Enoki, Phys. Rev. Lett. 78, 1335 (1997).
[5] G. Dresselhaus, Phys. Rev. 100, 580 (1955).
[6] E. I. Rashba, Sov. Phys. Solid State 2, 1109 (1960).
[7] P. Földi, V. Szaszkó-Bogár, and F. M. Peeters, Phys. Rev. B 82, 115302 (2010).
[8] P. Földi, V. Szaszkó-Bogár, and F. M. Peeters, Phys. Rev. B 83, 115313 (2011).
[9] V. Szaszkó-Bogár, P. Földi, and F. M. Peeters, J. Phys.: Cond. Matt. 26, 135302 (2014).

Thank you for your attention!