MOTT SKYRMIONS: STABILIZING THE FALSE VACUUM

Gergely Zaránd

Budapest University of Technology and Economics (BME)

Collaborators:

M. Kanász-Nagy and **B. Dóra** (BME)

E. Demler (Harvard)

M. Kanász-Nagy, B. Dóra, E. Demler, and G.Z., Scientific Reports 5, 7692 (2015)

Szeged, Elméleti Fizika Tanszék, March 2015

Mott syrmionok, avagy hogyan stabilizálhatunk topológikus gerjesztéseket erős kölcsönhatással

Gergely Zaránd

Budapest University of Technology and Economics (BME)

Collaborators:

M. Kanász-Nagy and **B. Dóra** (BME)

E. Demler (Harvard)

M. Kanász-Nagy, B. Dóra, E. Demler, and G.Z., Scientific Reports 5, 7692 (2015)

Szeged, Elméleti Fizika Tanszék, March 2015

OUTLINE

- **Introduction:**
  - **skyrmions as topological excitations**
  - **the optical toolbox**
  - **skyrmions in nematic superfluid**
- **Mott skyrmions**
  - **stabilizing the skyrmion**
  - **preparation**
  - **excitation spectrum**
  - **detection**
- **Conclusions**

INTRODUCTION

Topological excitations

Order parameter field: $\psi: \quad \underline{x} \quad \rightarrow \quad \psi(\underline{x}) \in \chi$

in space (space-time)

order parameter space

Homotopy classes: $\left\{ \underset{\underline{x} \in M}{\psi: \quad M \rightarrow \chi} \right\} / \left\{ \text{continuous deformations} \right\}$

non-trivial homotopy class structure $\Rightarrow$

- **topologically distinct field configurations**
- **topological excitations**

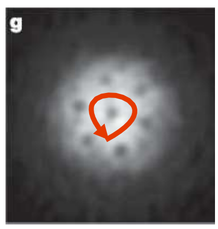
Topological defects / excitations

Vortices: structure of ψ on circle (first homotopy group $\Pi_1(\chi)$) is non-trivial

- superfluid vortices

$$\Pi_1(U(1)) = \Pi_1(S^1) = \mathbb{Z}$$

Zwierlein et al., Nature **435,** 1047 (2005)



- non-abelian vortices, Z_2 vortices etc.

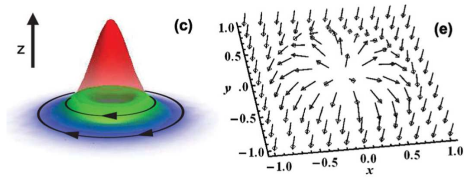
Non-trivial structure on **higher dimensional surfaces**: $\Pi_d(\chi) \neq \{0\}$

$\Rightarrow$ **monopoles** (defects)

't Hooft-Polyakov monopole in SU(2) field theories, e.g.
'Hedgehog' configurations in magnets

$\Rightarrow$

$$\Pi_2(S^2) = \mathbb{Z}$$



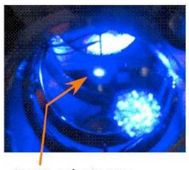
Leslie et al., PRL **103,** 250401 (2009)

Can one use cold atoms to create such topological objects?

The optical toolbox...

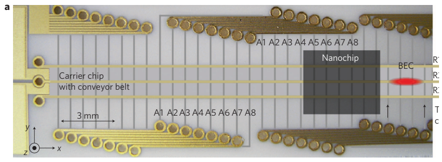
Trapping and cooling atoms:

Laser cooling
Magneto-optical trapping
Evaporative cooling
atomic chips
...



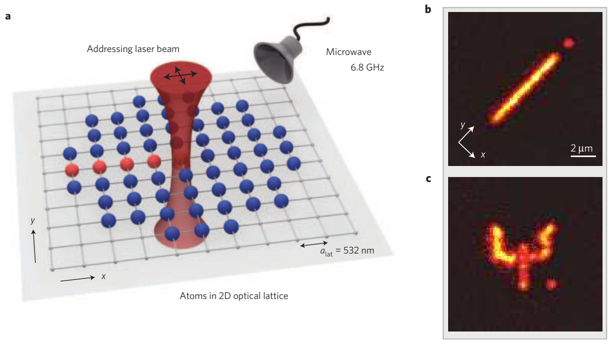
trapped atoms

$T < 1\mu\text{K}$



Creating potentials, interactions

Optical lattices
Feschbach resonances
low-dimensional structures
design potentials by holography!
single atom microscope and manipulation
....



[Weitenberg, …, S. Kuhr, Nature 471 (2011)]

Detection

Time of flight imaging
Bragg spectroscopy
Shaking, modulation spectroscopy
...

Monopoles and nematic order in spinor condenstates ?

^{23}Na: $F = 1$ $\Rightarrow$ $\psi = (\psi_x, \psi_y, \psi_z)$

Energy density $$H = \frac{1}{2m}|\nabla\psi|^2 - \mu\,|\psi|^2 + \frac{g_0}{2}\,|\psi|^4 + \frac{g_2}{2}\left(\psi^+\vec{F}\,\psi\right)^2$$

Nematic phase $g_2 > 0$

$\Rightarrow$ $\psi^+\vec{F}\,\psi = 0$ $\Rightarrow$ $\boxed{\psi = \underline{u}\sqrt{\rho_s}\,e^{i\phi}}$

real unit vector phase

Order parameter space $\chi = (S^2 \times U(1)) / Z_2$

$\Rightarrow$ $\Pi_2(\chi) = Z$ $\Rightarrow$ **monopoles (3D)... skyrmion in 2D**

T.-L. Ho, PRL **81**, 742 (1998); H. T. C. Stoof *et al.*, PRL **87**, 120407 (2001)

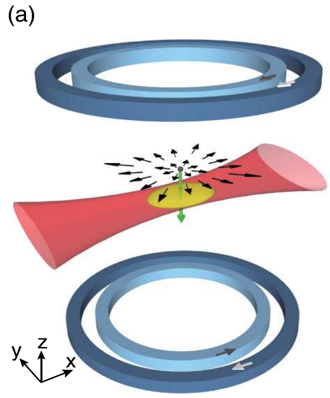
Trapping monopoles and skyrmions...

Solid state:
- Spin ice
- Skyrmion lattice in (helical) magnets (SO coupling!)
- Quantum-Hall effect

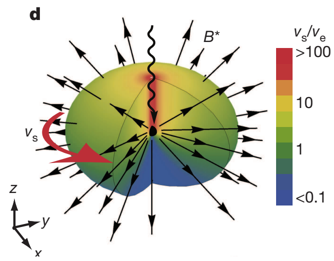
Cold atoms:
- Unstable superfluid textures succesfully created

Leslie et al, PRL 103, 250401 (2009).

Choi et al, PRL 108, 035301 (2012)
state collapses once field is removed



- Creation of ‘pseudomonopole’



imprinted, unstable

Ray et al, Nature 505, 657 (2014)

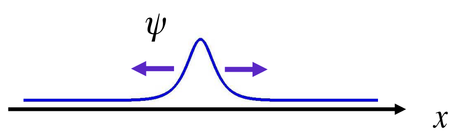
Skyrmions are generically UNSTABLE, they SHRINK or SLIP OUT of the trap...

Shrinkage: Derrick’s theorem

Skyrmions tend to shrink or expand...

Theorem: $E[\psi] = \int d^d x \quad \left\{ (\nabla\psi)^2 + f(\psi) \right\}$

Skyrmion unstable against shrinkage



proof:

$$\psi_\lambda(x) \equiv \psi(\lambda x) \qquad E_\lambda \equiv E[\psi_\lambda] = \lambda^{2-d} I_1 + \lambda^{-d} I_2$$

$\psi(x)$ local minimum $\Rightarrow \left(\frac{dE_\lambda}{d\lambda} \right)_{\lambda=1} = 0 \Rightarrow (d-2) I_1 + d\, I_2 = 0$

$\Rightarrow \left(\frac{d^2 E_\lambda}{d\lambda^2} \right)_{\lambda=1} < 0$ for $d > 2$

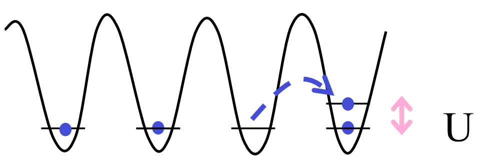
ways out:
- use external non-abelian gauge fields
- inhomogeneous potentials
- **compactify geometry !**

MOTT SKYRMION

Stabilizing the skyrmion by an optical lattice...

Mott transition for Bosons $\langle n_i \rangle = 1$

Bosons on an optical lattice:



$$\boxed{H = -J \sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j + \sum_i \epsilon_i \hat{n}_i + \frac{1}{2} U \sum_i \hat{n}_i (\hat{n}_i - 1)}$$

$\boxed{\langle n_i \rangle = 1}$

$U << J$ **Superfluid:**

$$|\Psi_{\mathrm{SF}}\rangle_{U=0} \propto \left(\sum_{i=1}^{M} \hat{a}_i^\dagger \right)^N |0\rangle$$

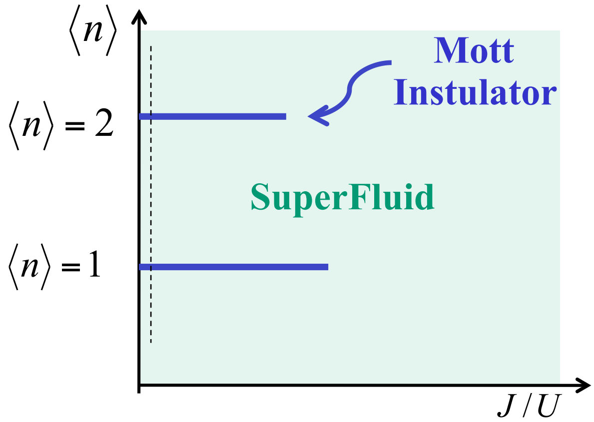
$$\langle SF | a_i \, a_0^+ | SF \rangle = n_s$$

$J << U$ **Insulator:**

$$|\Psi_{\mathrm{MI}}\rangle_{J=0} \propto \prod_{i=1}^{M} (\hat{a}_i^\dagger)^n |0\rangle$$

Phase diagram $\langle n_i \rangle \neq 1$

Phase diagram

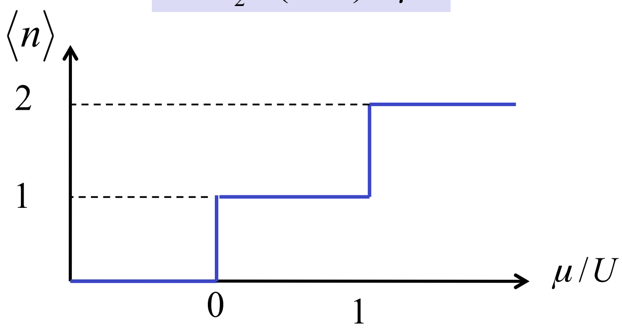


Mott Insulator is incompressible !

$J = 0$

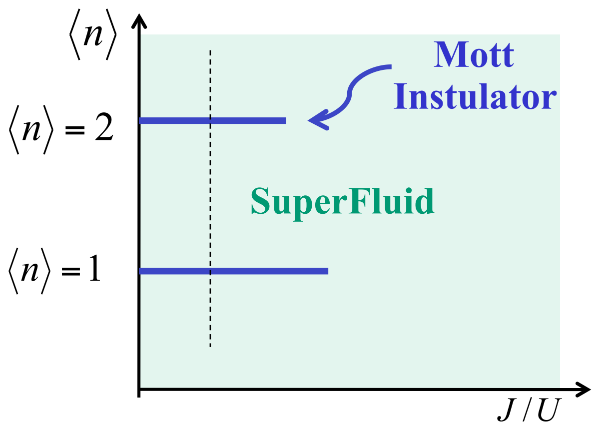
$T = 0$

$$H = \tfrac{U}{2} n(n-1) - \mu n$$



Phase diagram $\langle n_i \rangle \neq 1$

Phase diagram

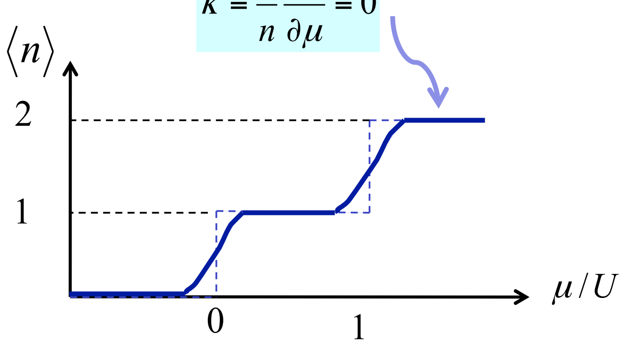


Mott Insulator is incompressible !

$J \neq 0$

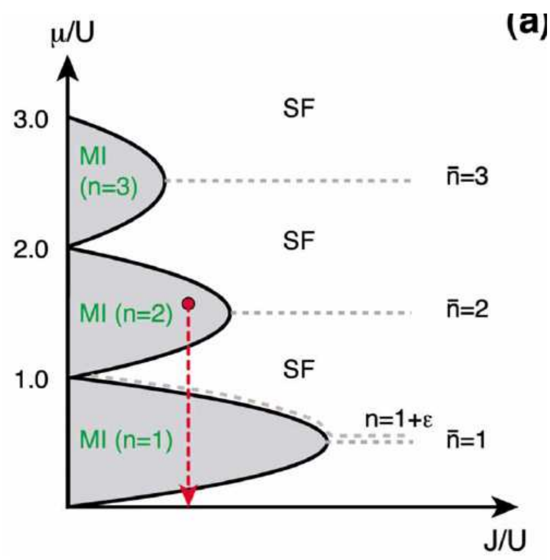
$T = 0$

$$\kappa = \frac{1}{n}\frac{\partial n}{\partial \mu} = 0$$



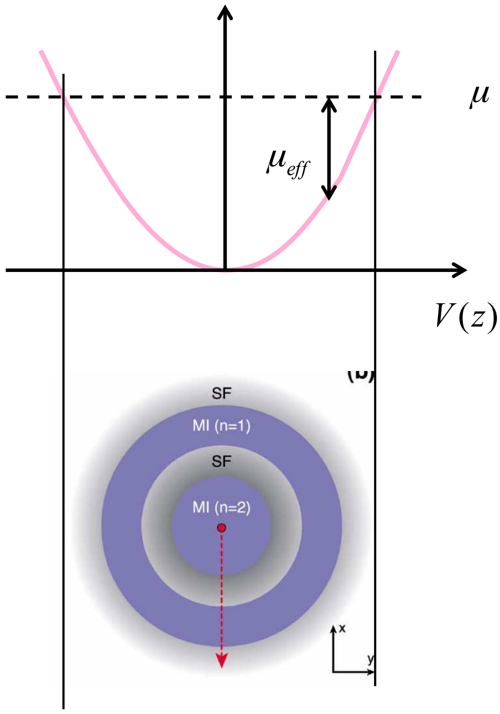
Phase diagram in a trap...

effective chemical potential varies across the trap !



Bloch, et al. RMP 2008

Bosons in a trap: Shell structure



Observation of bosonic Mott transition

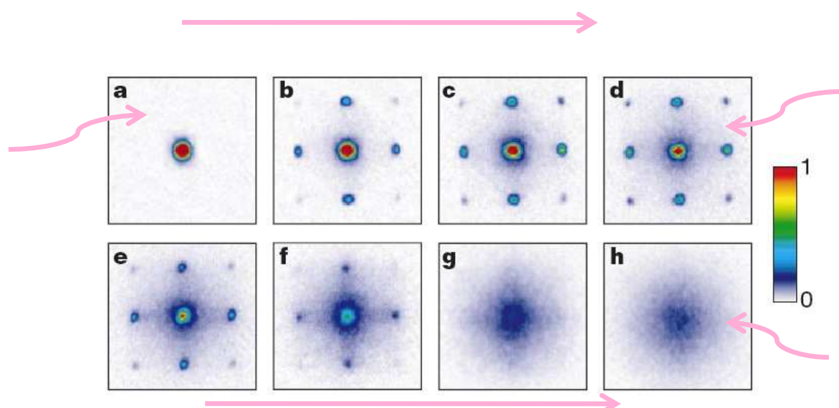
Interference pattern
Time of Flight experiment

^{87}Rb

Optical lattice

Condensate

Condensate with Bragg peaks



$\frac{U}{t} > \left(\frac{U}{t}\right)_C$

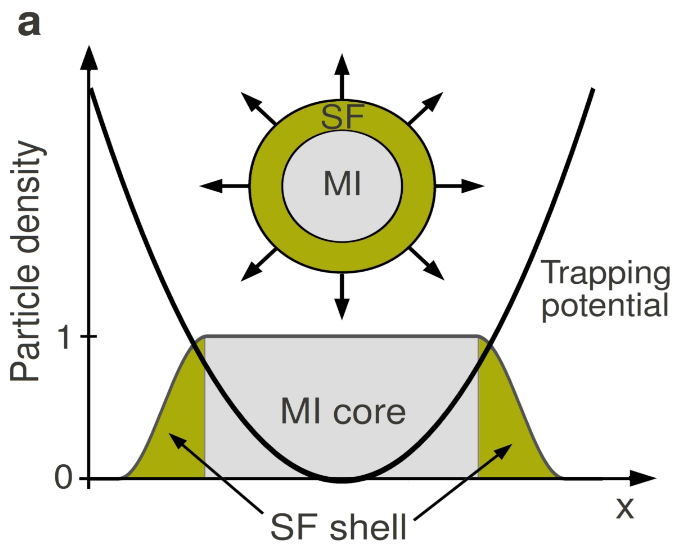
Mott insulator

Figure 2 Absorption images of multiple matter wave interference patterns. These were obtained after suddenly releasing the atoms from an optical lattice potential with different potential depths V_0 after a time of flight of 15 ms. Values of V_0 were: **a**, 0 E_r; **b**, 3 E_r; **c**, 7 E_r; **d**, 10 E_r; **e**, 13 E_r; **f**, 14 E_r; **g**, 16 E_r; and **h**, 20 E_r.

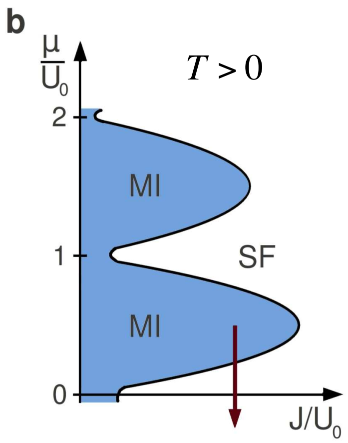
Greiner et al., Nature 415, 39 (2002)

Mott skyrmion

Optical lattice ⇨ Mott insulator core ⇨ **closed geometry + pinning!**



F = 1 Bose-Hubbard model



$$H_{\mathrm{kin}} = -J \sum_{\langle \mathbf{r},\mathbf{r}' \rangle} b^\dagger_{\mathbf{r}\,\alpha} b_{\mathbf{r}'\alpha},$$

$$H_{\mathrm{loc},\mathbf{r}} = -\mu(\mathbf{r}) n_{\mathbf{r}} + \frac{U_0}{2} : n_{\mathbf{r}}^2 : + \frac{U_2}{2} : \vec{F}_{\mathbf{r}}^2 : .$$

with $n_{\mathbf{r}} = \sum_\alpha b^\dagger_{\mathbf{r}\alpha} b_{\mathbf{r}\alpha}$ density

and $\vec{F}_{\mathbf{r}} = \sum_{\alpha,\beta} b^\dagger_{\mathbf{r}\alpha} \vec{F}_{\alpha\beta} b_{\mathbf{r}\beta}$ spin operators.

parameter range:

$U_2 << U_0$ $\quad$ $T << U_0$ $\quad$ $T >> T_C \sim z\, J^2 / U_0$

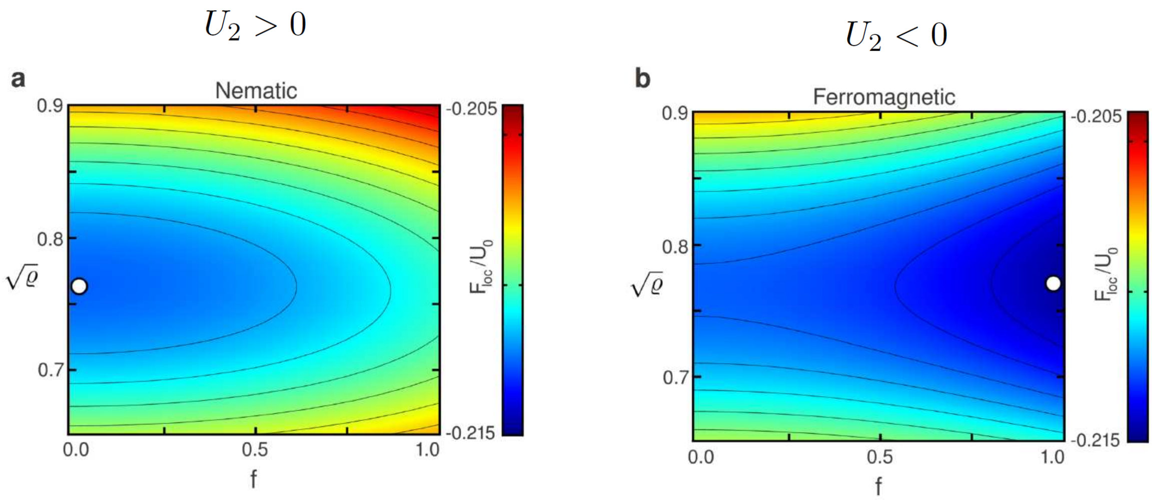
M. P. A. Fisher et al., PRB **40**, 546 (1989)
E. Demler and F. Zhou, PRL **88**, 163001 (2002)
F. Gerbier, PRL **99**, 120405 (2007)

Free energy functional

Hubbard-Stratonovich transformation (in hopping !) $b_{\mathbf{r}} \rightarrow \Psi_{\mathbf{r}}$

$$F(\{\Psi_{\mathbf{r}}\}) \approx -Ja^2 \sum_{\mathbf{r},\mathbf{r}',\alpha} \overline{\Psi}_{\mathbf{r}\alpha} \Delta_{\mathbf{r}\mathbf{r}'} \Psi_{\mathbf{r}'\alpha} + \sum_{\mathbf{r}} F_{\mathrm{loc}}\left(\varrho_{\mathbf{r}},\, \mathbf{f}_{\mathbf{r}}^2, \mu(\mathbf{r}), T\right)$$

Nematic interactions ($U_2 > 0$) $\Rightarrow$ $\boxed{\mathbf{f}_{\mathbf{r}} \equiv \Psi_{\mathbf{r}}^{\dagger} \vec{F} \Psi_{\mathbf{r}} / |\Psi_{\mathbf{r}}|^2 \equiv 0}$



Numerical minimazition

Artificial dynamics:

$$\left.\begin{aligned} -\partial_\tau \Psi_{\mathbf{r}\alpha} &= \frac{\delta F}{\delta \overline{\Psi}_{\mathbf{r}\alpha}} \\ -\partial_\tau \overline{\Psi}_{\mathbf{r}\alpha} &= \frac{\delta F}{\delta \Psi_{\mathbf{r}\alpha}} \end{aligned}\right\} \Rightarrow$$

Decreasing free energy: $\frac{dF}{d\tau} = -2 \sum_{\mathbf{r}\alpha} \left| \frac{\delta F}{\delta \Psi_{\mathbf{r}\alpha}} \right|^2 < 0$

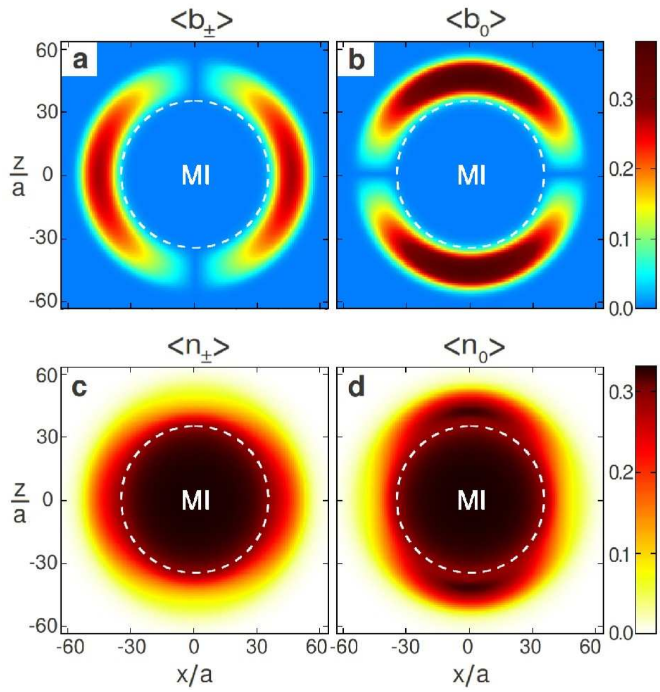
1. Start from a **hedgehog-like** solution
2. Add **noise**
3. Let it **relax**

Simulation parameters:

$$T/U_0 = 0.05,\ U_2/U_0 = 0.025,\ zJ/U_0 = 0.18,$$

$\Rightarrow$ $T \approx 10\, T_{C,\mathrm{magn}}$

Density cuts



Initializing the skyrmion

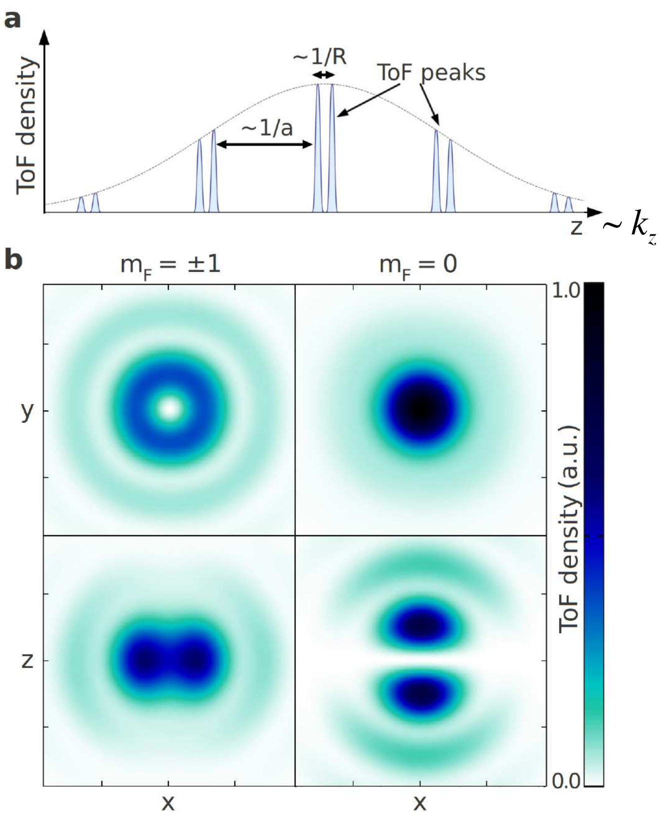
Spin basis: $\Psi_{m_F}(\mathbf{r}) = \begin{pmatrix} \Psi_+(\mathbf{r}) \\ \Psi_0(\mathbf{r}) \\ \Psi_-(\mathbf{r}) \end{pmatrix} = \sqrt{\rho(\mathbf{r})} \begin{pmatrix} \frac{\mathbf{x}+i\mathbf{y}}{\sqrt{2}} \\ \mathbf{z} \\ \frac{\mathbf{x}-i\mathbf{y}}{\sqrt{2}} \end{pmatrix} \begin{matrix} \longleftarrow \text{Vortex} \\ \longleftarrow \text{Dark soliton} \\ \longleftarrow \text{Antivortex} \end{matrix}$

$$\begin{pmatrix} \psi_1 \\ \psi_0 \\ \psi_{-1} \end{pmatrix} = |\psi| \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \xrightarrow[1.]{} |\psi| \begin{pmatrix} \frac{\hat{x}+i\hat{y}}{\sqrt{2}} \\ 0 \\ |\hat{z}| \end{pmatrix} \xrightarrow[2.]{} |\psi| \begin{pmatrix} 0 \\ \frac{\hat{x}+i\hat{y}}{\sqrt{2}} \\ |\hat{z}| \end{pmatrix} \xrightarrow[3.]{} |\psi| \begin{pmatrix} \frac{\hat{x}-i\hat{y}}{\sqrt{2}} \\ \frac{\hat{x}+i\hat{y}}{\sqrt{2}} \\ |\hat{z}| \end{pmatrix} \xrightarrow[4.]{} |\psi| \begin{pmatrix} \frac{\hat{x}-i\hat{y}}{\sqrt{2}} \\ \frac{\hat{x}+i\hat{y}}{\sqrt{2}} \\ \hat{z} \end{pmatrix}$$

1. Create vortex: Gaussian + Laguerre-Gaussian beam (G, LG^+)
2. RF shift
3. Create anti-vortex (G, LG^-)
4. Imprint dark soliton

K. C. Wright et al.,PRA **78**, 053412 (2008).
L. S. Leslie et al., PRL **103**, 250401 (2009).

Time of flight



$$\boxed{n_\alpha^{\mathrm{ToF}} \propto C_\alpha\left(\mathbf{k} = \frac{m\mathbf{r}}{t}\right)}$$

with an in-trap correlation function

$$C_\alpha(\mathbf{k}) \approx \left|\sum_{\mathbf{r}} \langle \Psi_{\mathbf{r}\alpha} \rangle e^{i\mathbf{kr}}\right|^2 + const.$$

Excitation spectrum

Effective model on the **superfluid shell**

$$\boxed{T = i\psi^+\partial_t\psi - \left[-\psi^+\left(\frac{\Delta}{2m} + \mu\right)\psi + \frac{g_0}{2}|\psi|^4 + \frac{g_2}{2}\left(\psi^+\vec{F}\psi\right)^2\right]}$$

$\Rightarrow$ $$i\,\partial_t\psi = \left(-\frac{\Delta_2}{2m} - \tilde{\mu} + g_0|\psi|^2\right)\psi + g_2(\overline{\psi}\vec{F}\psi)\cdot\vec{F}\psi.$$

Trivial (homogeneous) configuration: $\psi \sim e^{i\phi}\,\hat{z}$

Fluctuations: $\delta\psi = \delta\psi_\perp + \delta\psi_\parallel$

spin charge/phase

- **charge and spin modes decouple**
- **excitation frequencies**

$$\omega_{\mathrm{sp},l} \approx \frac{1}{mR\xi_2}\sqrt{l(l+1)} \qquad \omega_{\mathrm{ph},l} \approx \frac{1}{mR\xi_0}\sqrt{l(l+1)}$$

$g_2 << g_0$ $\Rightarrow$ $\xi_2 >> \xi_0$

$\Rightarrow$ **low-energy excitations live in spin sector !**

Exitation spectrum

Skyrmion configuration: $\psi \sim e^{i\phi}\,\underline{\hat{r}}$

non-abelian **gauge field**

Trivial		**Skyrmion**
Δ_2	$\longrightarrow$	$(\nabla + i\mathbf{A})^2 + \frac{2}{R^2}$

Curvature term (depletion of SF)

gauge field

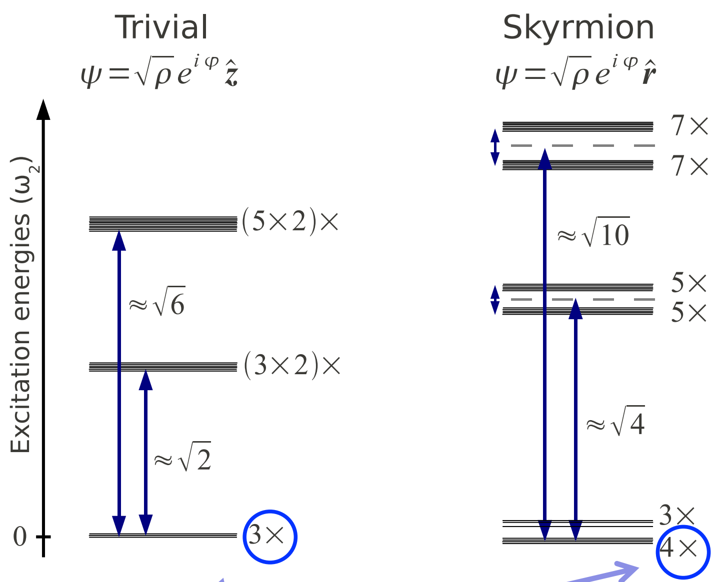
- couples spin and charge
- cannot be gauged away
  ~ monopole within the sphere

⇨ **chages spectrum**

expand $\delta\psi$ in (vector) spherical functions

Exitation spectrum

typical energy $\omega_2 = 1/(mR\xi_2)$ ~ 5-10 Hz.



spin degeneracy removed

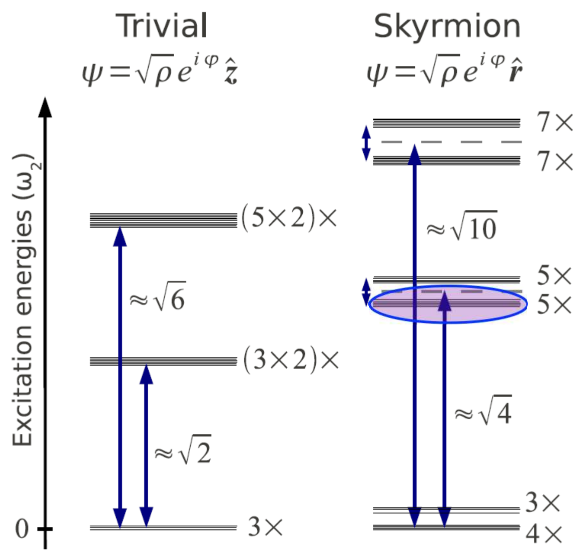
Spontaneous distortion of skyrmion.

Goldstone modes: 1 phase mode, **unphysical !**
2 (+1) spin modes ⟷ **Anderson's tower**

Modulation spectroscopy

Modulation of the hopping in the **z** direction $\Longleftrightarrow$ ∂_z^2 in the continuum model.

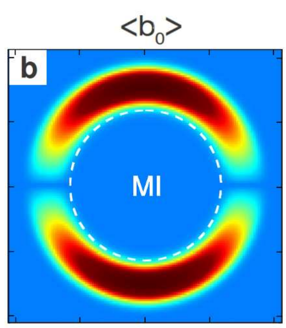
Selection rules ⇨



CONCLUSIONS

- **Skyrmion stabilized** by Mott insulating core
- Straightforward skyrmon **creation protocols**
- Characteristic **in trap** and **time of flight images**
- Excitation **spectrum** reflects **topology**

⇨ Modulation spectroscopy



M. Kanász-Nagy, B. Dóra, E. Demler, and G. Z., Scientific Reports 5, 7692 (2015)