Kvantum-optikai módszerek a nagyenergiás fizikában

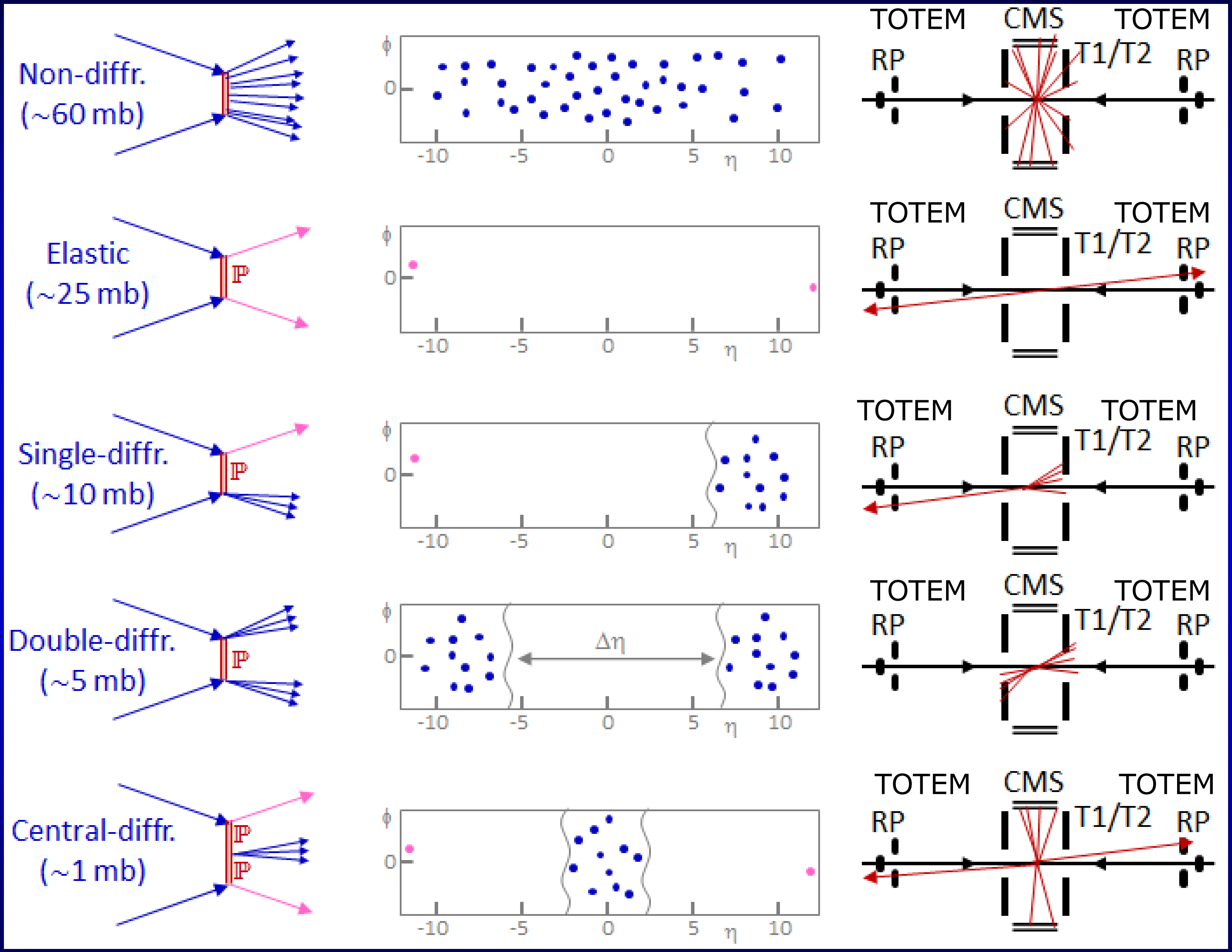
Csörgő T. [1,2]

[1] MTA Wigner FK, Budapest és
[2] KRF, Gyöngyös

Rugalmas p+p @ 7 TeV LHC
Pomeron Femtoszkópia a la Bialas-Bzdak
A többszörös diffrakció Glauber-Velasco elmélete
Korrelációs függvények modell független elemzése

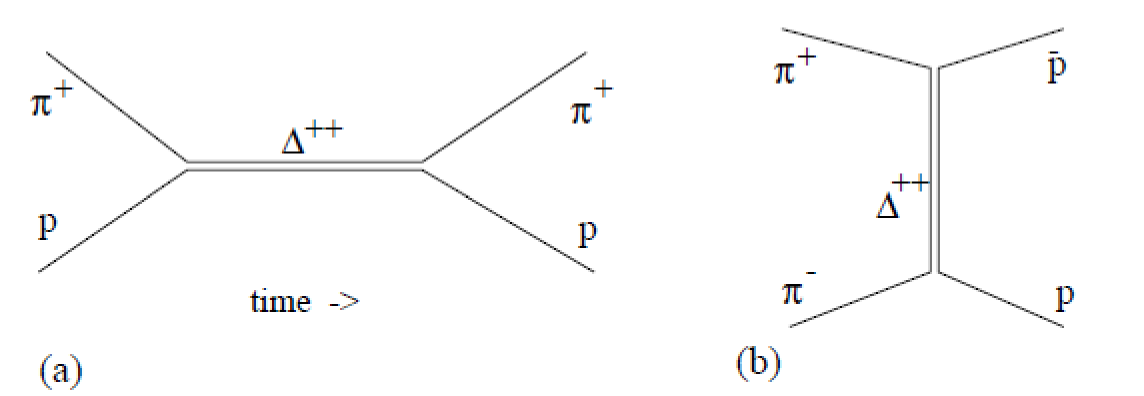
arXiv:1204.5617
arXiv:1306.4217
arXiv:1311.2308
arxiv:1505.01415
R. J. Glauber @ WPCF 2014, Cs. T. és Novák T. @ WPCF 2015
+ kéziratok előkészületben

Pomeron fizika: CERN LHC TOTEM



Rugalmas és diffractív szórás: Pomeron (színtelen) csere

Bevezetés a Pomeron fizikába



Crossing szimmetria:
s-csatornában részecske-keltés: (a)
t-csatornában kölcsönhatás részecske cserével: (b)

$$A(s,t) = 16\pi \sum_{l=0}^{\infty} (2l+1) a_l(s) P_l(\cos\theta),$$

$$a_l \sim 1/(s - m_l^2 + i m_l \Gamma_l)$$

$$A(s,t) = 16\pi \sum_{l} (2l+1) a_l(t) P_l(z_t),$$

$$a_l(t) \sim 1/(t - m_l^2 + i m_l \Gamma_l)$$

$$z_t = \cos\theta_t = \frac{2s}{t - s_0} + 1$$

R. Engel, hep-ph/0111396
E. Levin, hep-ph/9808486

s, t változók, rugalmas pp:

$$p(p_1) + p(p_2) \to p(p_3) + p(p_4)$$
$$s = (p_1 + p_2)^2,\ t = (p_1 - p_3)^2$$

$$s_0 \sim 1\mathrm{GeV}^2$$

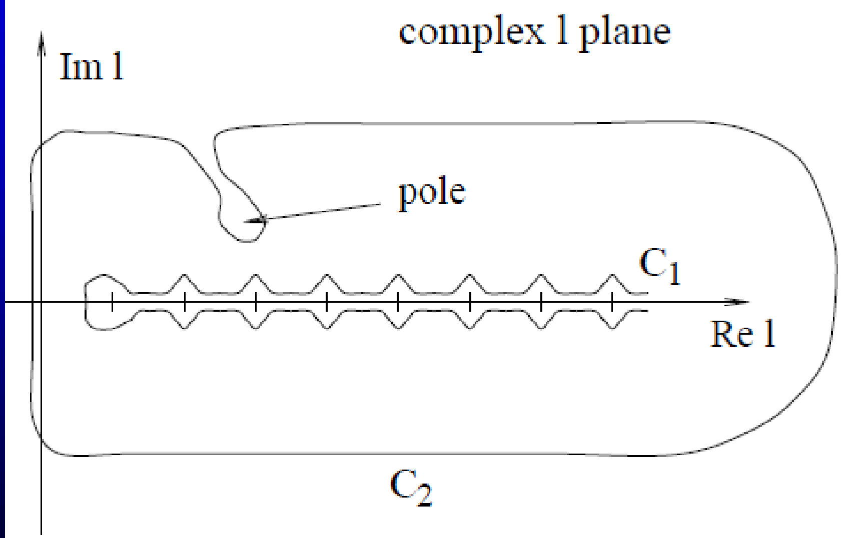
Regge elmélet

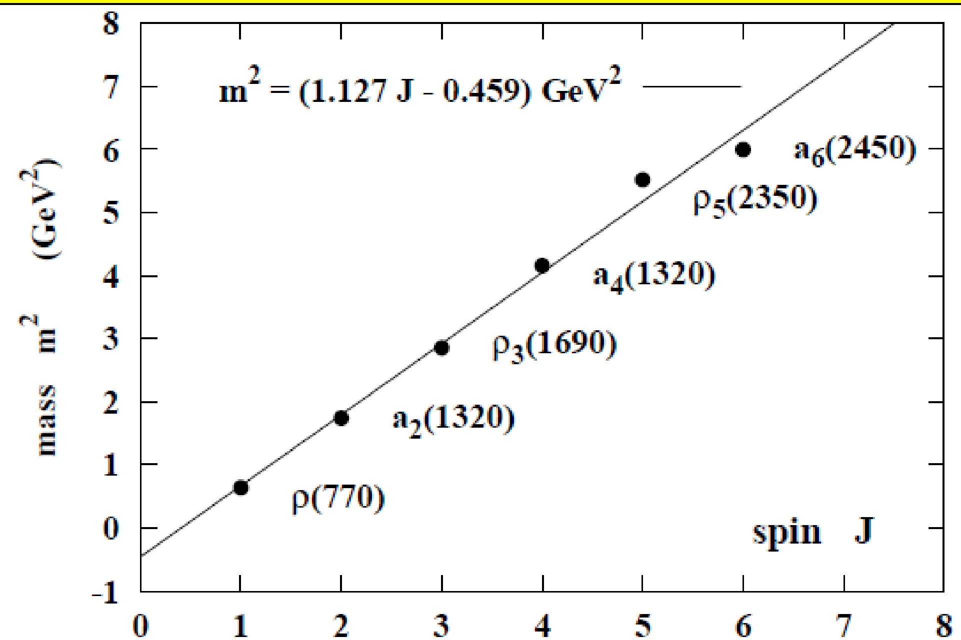
$$A(s,t) = \sum_{\tau=\pm1} \frac{16\pi}{2i} \int_{C_1} dl\,(2l+1)\left(\frac{1+\tau e^{-i\pi l}}{\sin(\pi l)}\right) a_l(t)\, P_l(-z_t), \qquad \tau = \pm 1$$

$$a_l \sim \frac{1}{t^2 - m_l^2} = \frac{1}{t - m_0^2 - al} \sim \frac{1}{l - t/a + m_0^2/a} = \frac{1}{l - \alpha(t)}$$

$$m_l^2 = al + m_0^2$$

Regge trajektóriák,
Pólus helye $\alpha(t) = l$, $\alpha(t) = \alpha(0) + \alpha'(0)\, t$





Reggeons

$$A(s,t) = -\frac{1+\tau e^{-i\pi\alpha(t)}}{\sin(\pi\alpha(t))}\beta(t)P_{\alpha(t)}(-z_t)$$

$$P_{\alpha_k(t)}\left(-\frac{2s}{t-s_0}-1\right) \xrightarrow{s\to\infty} \left(\frac{s}{s_0}\right)^{\alpha_k(t)}$$

$$A(s,t) = \sum_k \eta(\alpha_k(t))\beta_k(t)\left(\frac{s}{s_0}\right)^{\alpha_k(t)}$$

$$\eta(\alpha_k(t)) = -\frac{1+\tau e^{-i\pi\alpha_k(t)}}{\sin(\pi\alpha_k(t))}$$

Summation is over Regge trajectories,
Reggeons = quasi-particle =
family of resonances with same quantum numbers

T. Regge, Nuovo Cim. 14 (1959) 951

Pomeronok és hatáskeresztmetszetek

$$\frac{d\sigma_{\rm ela}}{dt} = \frac{1}{16\pi s^2} |A(s,t)|^2$$

$\sigma_{\rm tot}$ … 1

$$\Delta = \alpha(0) - 1$$

$$\sigma_{\rm ela} = (1+\rho^2)\frac{g^2}{16\pi}\left(\frac{s}{s_0}\right)^{2\Delta} \exp\{B_{\rm ela} t\}$$

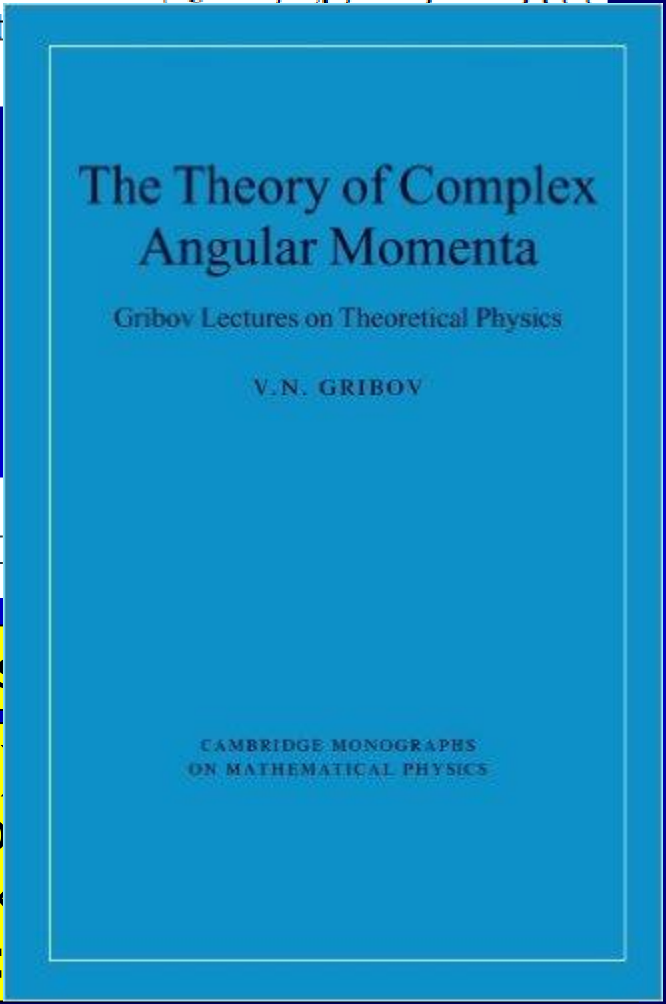
$$\sigma_{\rm tot} = g\left(\frac{s}{s_0}\right)^{\Delta}$$

$B_{\rm ela} = B$…

Az adatokból: $\sigma_{\rm tot}(s)$ ~ konst majd növeks…

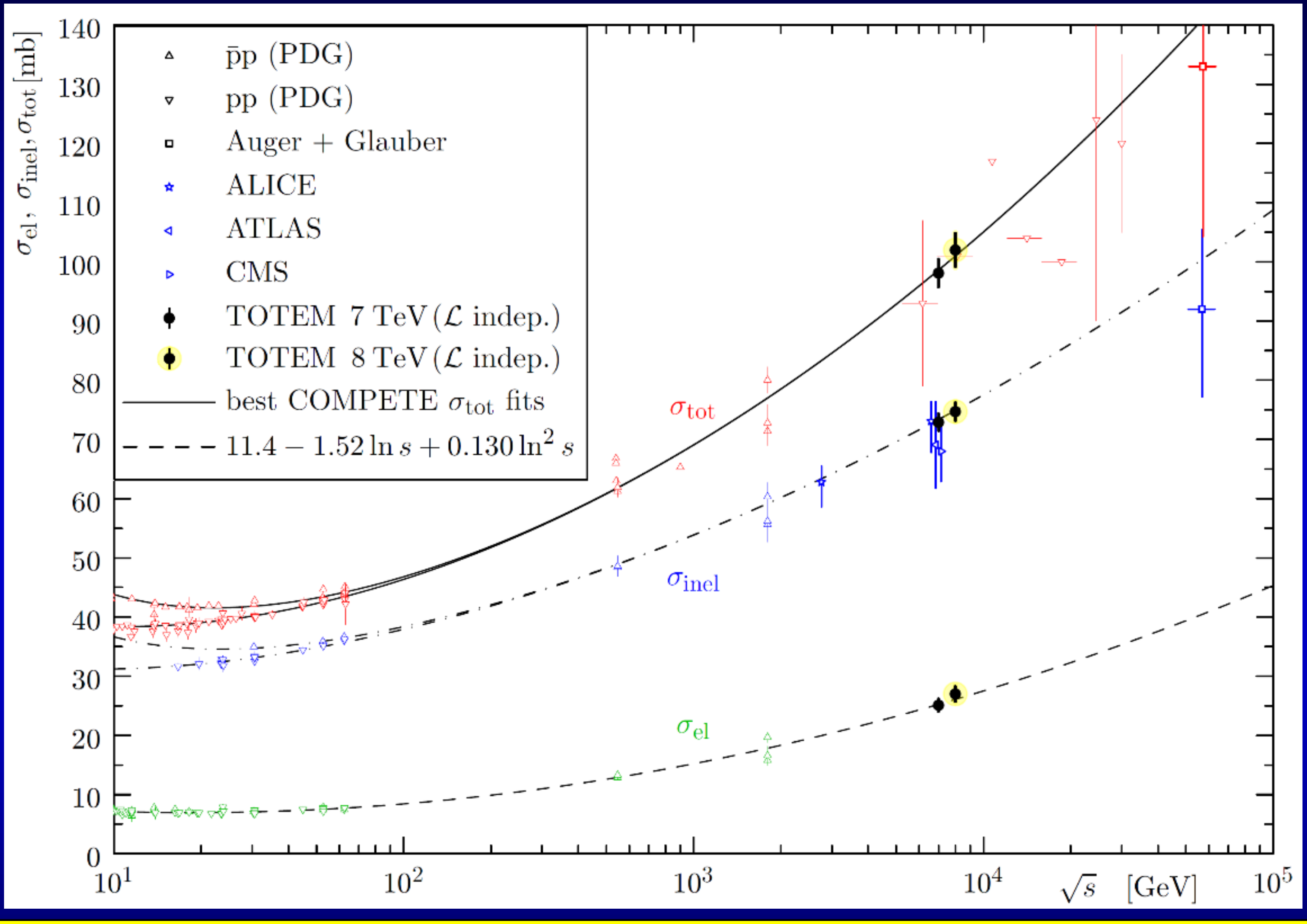
$\alpha(0) \gtrsim 1$

Pomeron (Pomeranchuk-on)…
Nincs ismert rezonancia a Po…
(vákuum kvantum-számok…
Ismert részecskékre:…



V. N. Gribov and I. Ya. Pomeranchuk, Phys. Rev. Lett. 8, 343

Pomeronok és hatáskeresztmetszetek



LHC adatokból: $\sigma_{tot}(s)$ növekvő, ~ ln s

Froissart-M. felső korlát: $\sigma_{tot}(s) \le C \ln^2 s$, Pomeronok ált. sértik

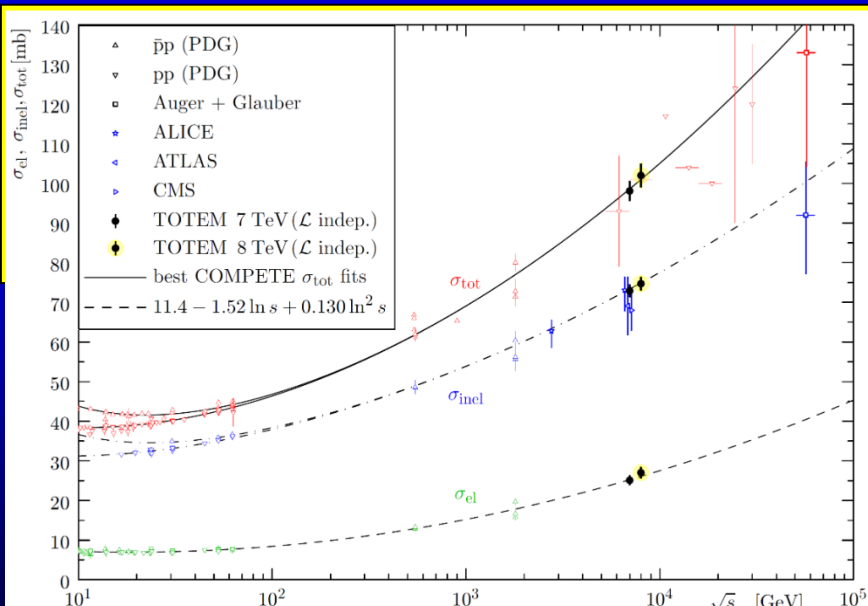
Gauss b-ben: Pomeron t-ben!

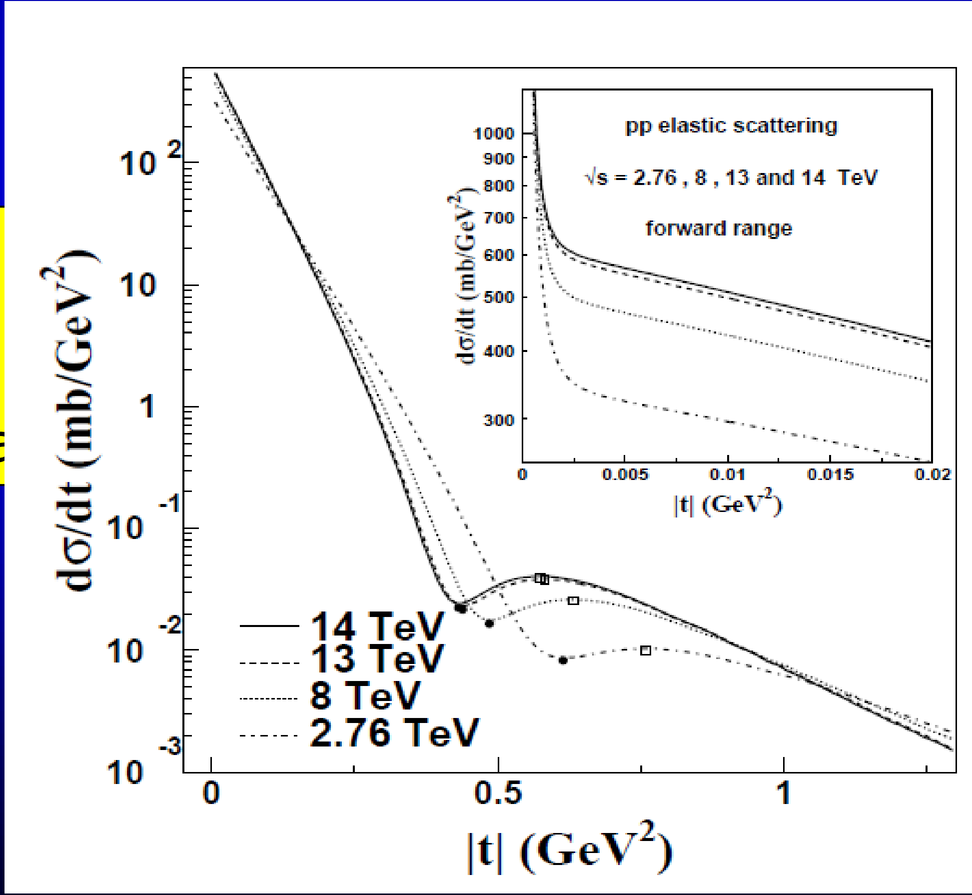
$$g_1(0)\, g_2(0) (s/s_0)^{\Delta_P} \frac{1}{\pi R^2(s)} e^{-\frac{b_t^2}{R^2(s)}}$$

Gauss közelítés:
Rugalmas szórási amplitúdó az ütközési paraméter (b) térben

$$\sigma_{tot} = \sigma_{el} + \sigma_{in} = 2\pi R^2$$

$$R^2(s) = R_0^2 + 4\,\alpha'_P \ln(s/s_0)$$





S-matrix unitaritás, optikai tétel

$$SS^\dagger = I\,,$$

$$S = I + iT$$

$$T - T^\dagger = iTT^\dagger$$

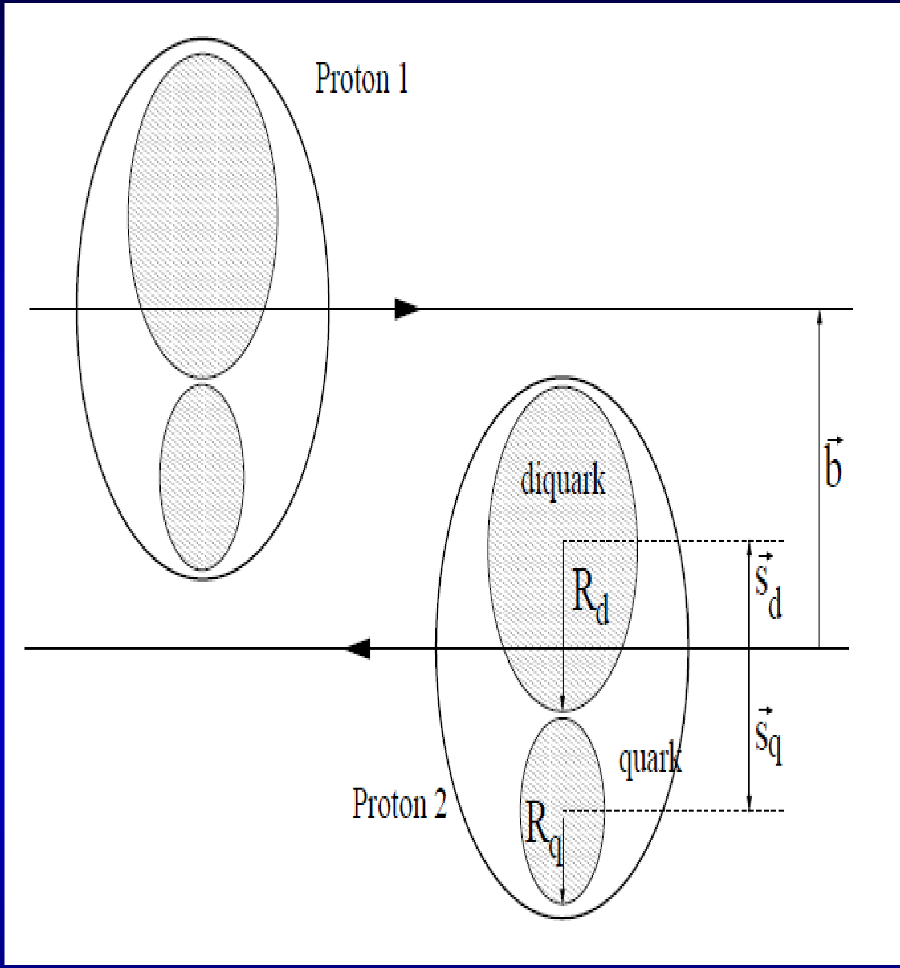
$$2\,\mathrm{Im}\,t_{el}(s,b) = |t_{el}(s,b)|^2 + \sigma(s,b)$$

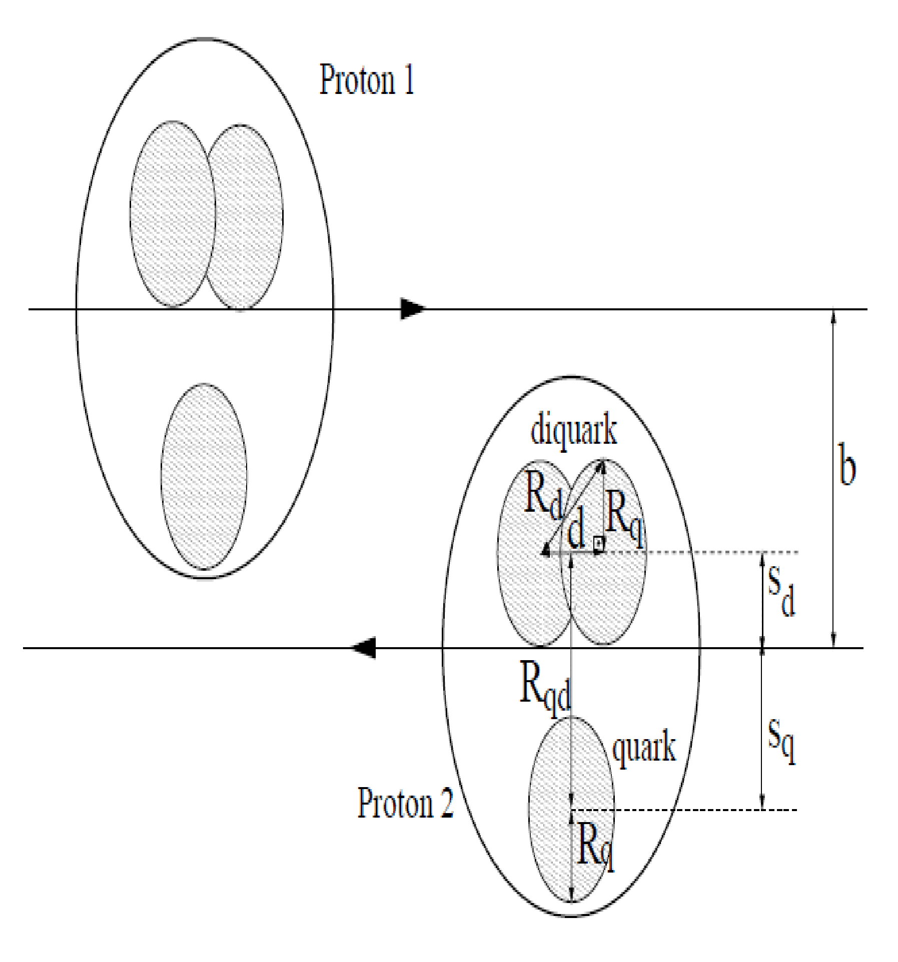
Megjegyzés: Rugalmas szórásokra |Fourier-transzf|^2 ~ dσ/dt

- Femtoszkópia lehetősége
- Inverz képalkotási probléma
- nem-Gauss források stb vizsgálatának lehetősége

Fekete (szürke) korong határeset (fontos, idealizált eset)
→ σ(b) ~ θ(R-b)

Diffraktív proton-proton szórás





p = (q, d)

p = (q, (q,q))

Bialas-Bzdak kvark-dikvark modell

$$\frac{d\sigma}{dt} = \frac{1}{4\pi} \left| T\left(\Delta\right) \right|^2 .$$

A. Bialas and A. Bzdak,
Acta Phys. Polon. B 38 (2007) 159
p= (q, d) vagy p = (q, (q, q))

$$T(\vec{\Delta}) = \int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty} t_{el}(\vec{b}) e^{i\vec{\Delta}\cdot\vec{b}} \mathrm{d}^2 b = 2\pi \int\limits_{0}^{+\infty} t_{el}\left(b\right) J_0\left(\Delta b\right) b \mathrm{d}b,$$

$$t_{el}(\vec{b}) = 1 - \sqrt{1 - \sigma(\vec{b})}.$$

σ(b) = b függő kölcsönhatási vszség elo.
→ képalkotási lehetőség a szórásról

$$\sigma(\vec{b}) = \int\limits_{-\infty}^{+\infty} \ldots \int\limits_{-\infty}^{+\infty} \mathrm{d}^2 s_q \mathrm{d}^2 s_q' \mathrm{d}^2 s_d \mathrm{d}^2 s_d' D(\vec{s_q}, \vec{s_d}) D(\vec{s_q}', \vec{s_d}') \sigma(\vec{s_q}, \vec{s_d}; \vec{s_q}', \vec{s_d}'; \vec{b}),$$

A protonok szerkezete = ?
→ Diffraktív pp szórás, ISR (23.5–62.5 GeV) és LHC (7-13 TeV).

Diffrakció a la Bialas és Bzdak

$$D\left(\vec{s_q},\vec{s_d}\right)=\frac{1+\lambda^2}{\pi R_{qd}^2}e^{-(s_q^2+s_d^2)/R_{qd}^2}\delta^2(\vec{s_d}+\lambda\vec{s_q}),\ \lambda=m_q/m_d,$$

$$\sigma(\vec{s_q},\vec{s_d};\vec{s_q}',\vec{s_d}';\vec{b})=1-\prod_{a,b\in\{q,d\}}\left[1-\sigma_{ab}(\vec{b}+\vec{s_a}'-\vec{s_b}')\right]$$

$$\sigma_{ab}\left(\vec{s}\right)=A_{ab}e^{-s^2/R_{ab}^2},\ R_{ab}^2=R_a^2+R_b^2,$$

Bialas és Bzdak kvark-dikvark modellje analitikusan integrálható a **Gauss közelítésben**, ***feltéve*** hogy a szórási amplitúdó valós része elhanyagolható.

Két kép: p = (q, d) vagy p = (q, (q,q))

Megj: p= (q,q,q) modell már az ISR-nál rossz, p ≠ (q,q,q)
W. Czyz and L. C. Maximon, Annals. Phys. 52 (1969) 59
„Rugós" p=(q,d) Pomeron modell, Grichine, arxiv:1404.5768

A BB modell valós kiterjeszése

$$\frac{d\sigma}{dt} = \frac{1}{4\pi}\left|T\left(\Delta\right)\right|^2.$$

$$T(\vec{\Delta}) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} t_{el}(\vec{b})e^{i\vec{\Delta}\cdot\vec{b}}\mathrm{d}^2b = 2\pi\int_{0}^{+\infty} t_{el}\left(b\right)J_0\left(\Delta b\right)b\mathrm{d}b,$$

$$t_{el}(s,b) = i\left(1 - e^{-i\,\mathrm{Im}\,\Omega(s,b)}\sqrt{1-\sigma(s,b)}\right)$$

Bialas-Bzdak eredeti,
ha Re (t_{el}) = 0

$$t_{el}(s,b) = i\left(1 - e^{-\mathrm{Re}\,\Omega(s,b)}\right) = i\left(1 - \sqrt{1-\sigma(s,b)}\right)$$

$$\sigma(\vec{b}) = \int_{-\infty}^{+\infty}\dots\int_{-\infty}^{+\infty} \mathrm{d}^2s_q\mathrm{d}^2s'_q\mathrm{d}^2s_d\mathrm{d}^2s'_d D(\vec{s_q},\vec{s_d})D(\vec{s_q}',\vec{s_d}')\sigma(\vec{s_q},\vec{s_d};\vec{s_q}',\vec{s_d}';\vec{b}),$$

A képzetes t_{el} valós kiterjesztése,
az opacitás függény képzetes része, Im Ω bevezetésével

ReBB: a BB modell valós kiterjesztése

$$\sigma(b) = \int_{-\infty}^{+\infty} \dots \int_{-\infty}^{+\infty} \mathrm{d}^2 s_q \mathrm{d}^2 s'_q \mathrm{d}^2 s_d \mathrm{d}^2 s'_d D(\mathbf{s}_q, \mathbf{s}_d) D(\mathbf{s}'_q, \mathbf{s}'_d)\, , \sigma(\mathbf{s}_q, \mathbf{s}_d; \mathbf{s}'_q, \mathbf{s}'_d; \mathbf{b})\, .$$

$$D\left(\mathbf{s}_q, \mathbf{s}_d\right) = \frac{1+\lambda^2}{R_{qd}^2 \pi} e^{-(s_q^2+s_d^2)/R_{qd}^2} \delta^2(\mathbf{s}_d + \lambda\, \mathbf{s}_q), \;\; \lambda = \frac{m_q}{m_d}\, ,$$

$$\sigma(\mathbf{s}_q, \mathbf{s}_d; \mathbf{s}'_q, \mathbf{s}'_d; \mathbf{b}) = 1 - \prod_{a,b \in \{q,d\}} \left[1 - \sigma_{ab}(\mathbf{b} + \mathbf{s}'_a - \mathbf{s}_b)\right]$$

$$\sigma_{ab}\left(\mathbf{s}\right) = A_{ab} e^{-s^2/R_{ab}^2}, \;\; R_{ab}^2 = R_a^2 + R_b^2, \;\; a, b \in \{q, d\}$$

$$\sigma_{qq} : \sigma_{qd} : \sigma_{dd} = 1 : 2 : 4$$

Bialas-Bzdak modell „realizálva”:
p = (q,d)
p= (q, (q,q))

Gauss b-ben,
Pomeron t-ben?

ReBB modell: két út

$$\mathrm{Im}\,\Omega(s,b) = -\alpha \cdot \mathrm{Re}\,\Omega(s,b)\,.$$

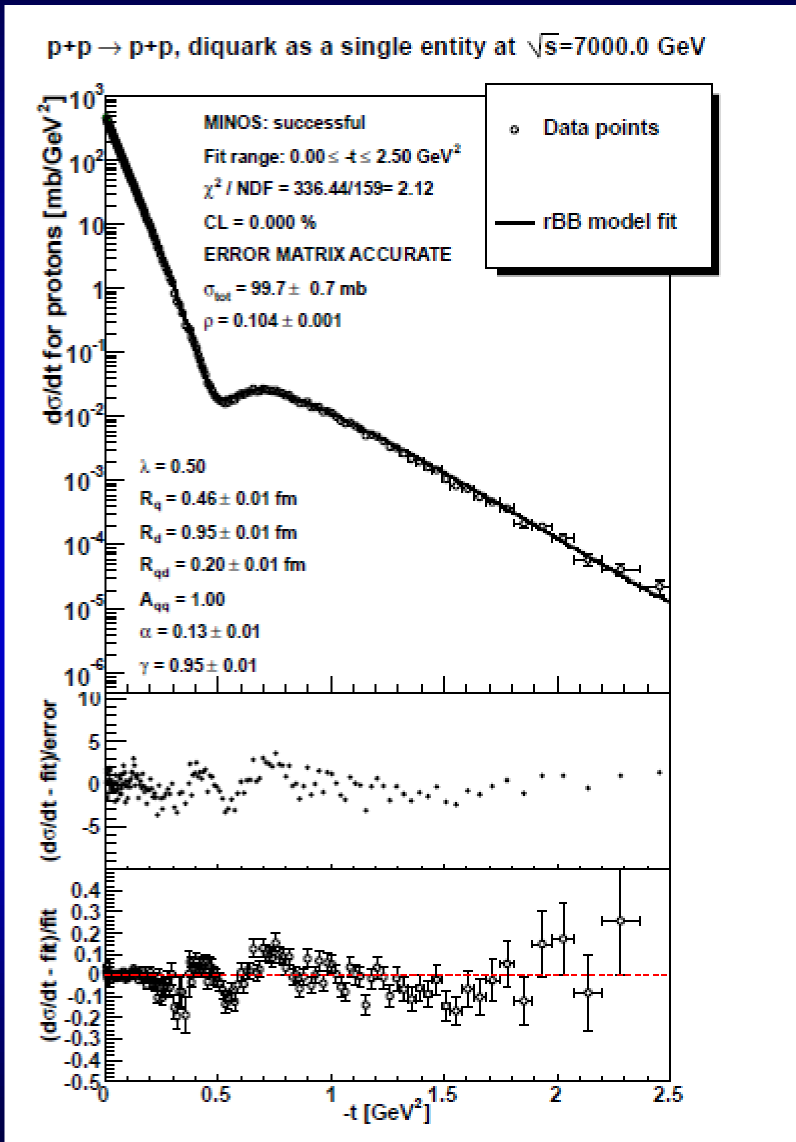
$$\mathrm{Im}\,\Omega(s,b) = -\alpha \cdot \tilde{\sigma}_{inel}(s,b)\,,$$

Hasonló idea:
ρ
állandó
- nincs
összhangban
az adatokkal

Kis
α
értékekre
visszakapjuk
az
αBB modellt
(ISR-on jó,
LHC-nél nem)

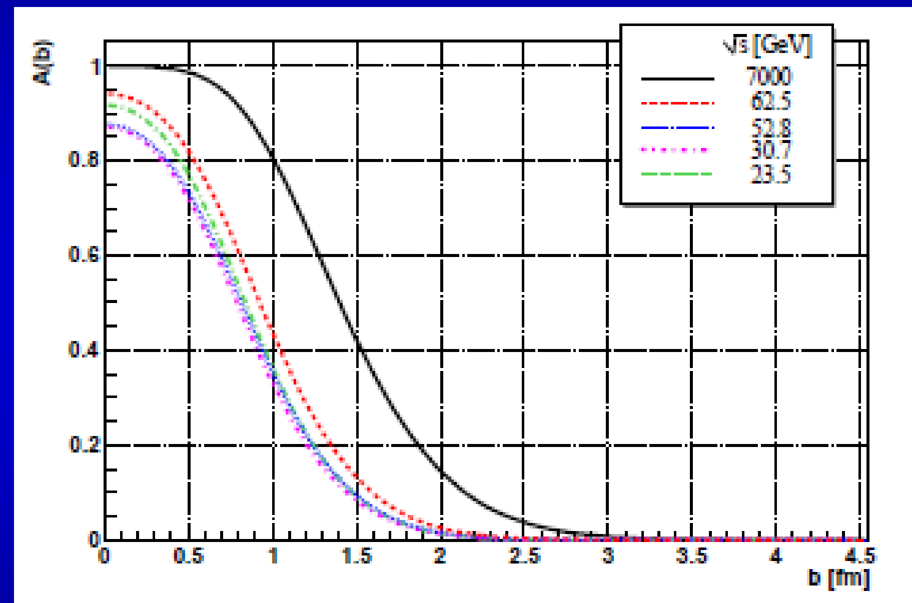
Az adatok a második lehetőségre utalnak
T. Cs., F. Nemes, arxiv:1306.4217

ReBB modell, két TOTEM adatsoron



Árnyékolási profil

$$A(s,b) = 1 - \left|\exp\left[-\Omega(s,b)\right]\right|^2$$

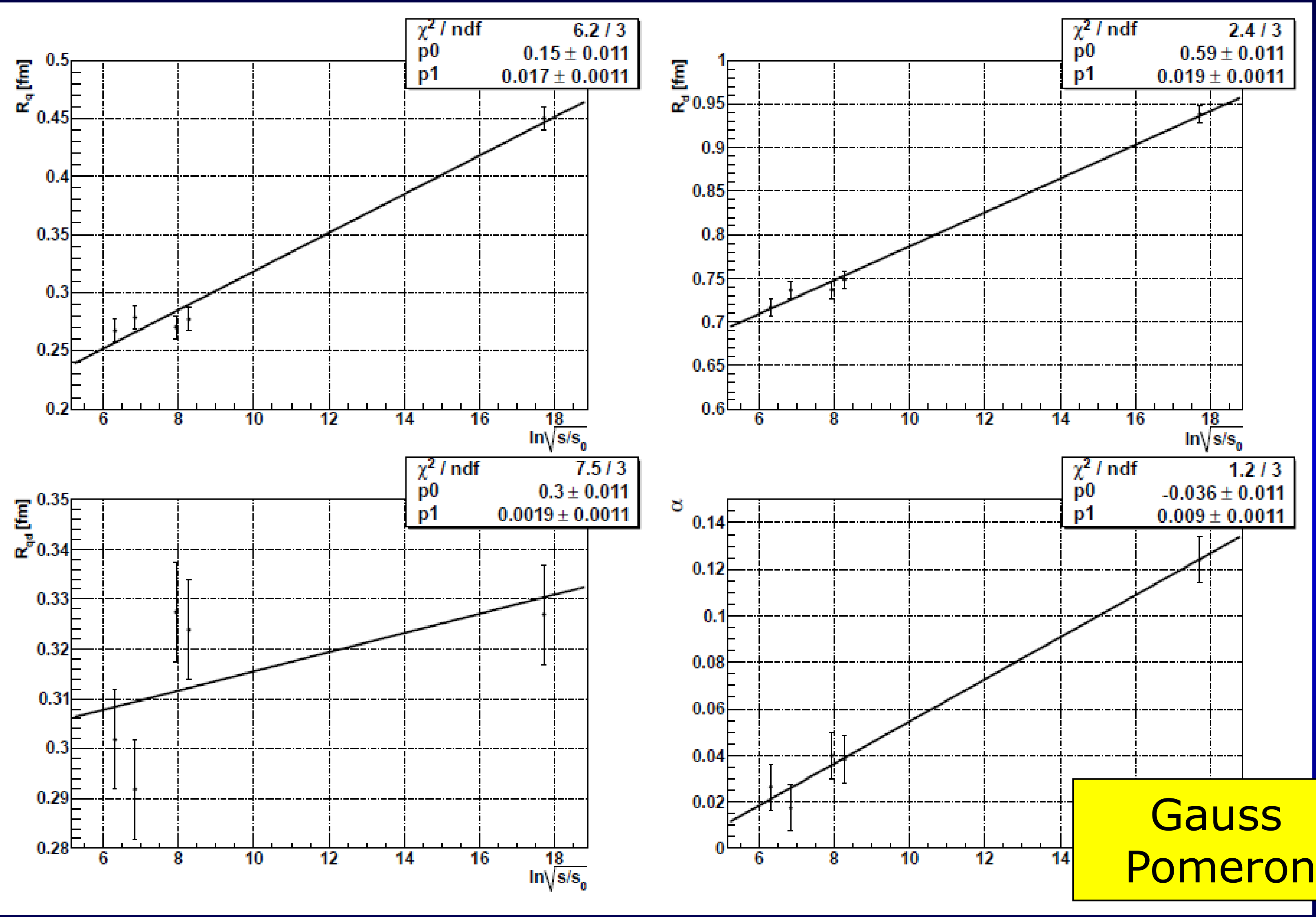


$$\frac{d\sigma}{dt} \to \gamma \cdot \frac{d\sigma}{dt}$$

$$t_{sep} = -0.375\ \mathrm{GeV}^2$$

Illesztés: $0 \le -t \le 2.5\ \mathrm{GeV}^2$,
nem teljesen OK → @ 8 TeV?

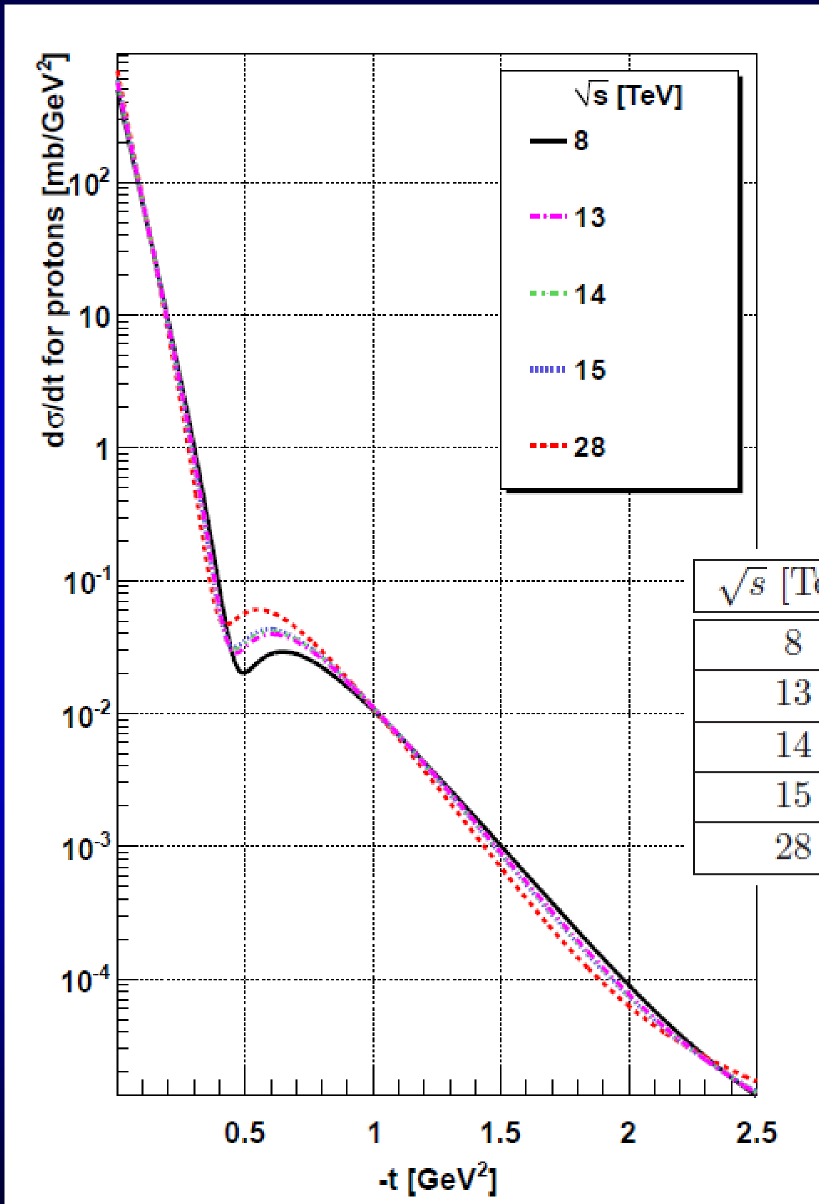
Gerjesztési függvény: pp skálázás



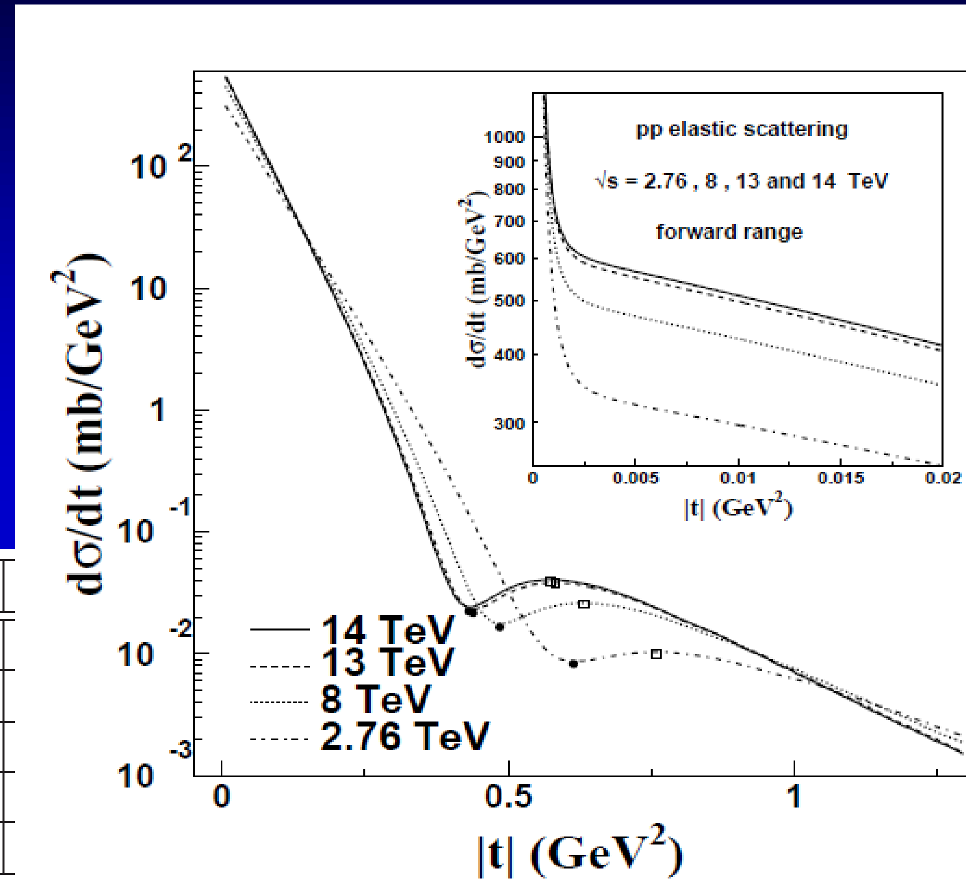
Gauss b-ben
Pomeron t-ben!

Geometriai skálázás: $\{R_q, R_d, R_{qd}, \alpha\} = p_0 + p_1 \ln (s/s_0)$

Gerjesztési függvény: dσ/dt



$\sqrt{s}$ [TeV]
8
13
14
15
28



de $t_{dip}\,\sigma_{tot}$ ~ const (2 %)
Hasonló eredmények:
K.A. Kohara, T. Kodama, E. Ferreira,
arXiv:1411.3518

ReBB árnyékolási profil

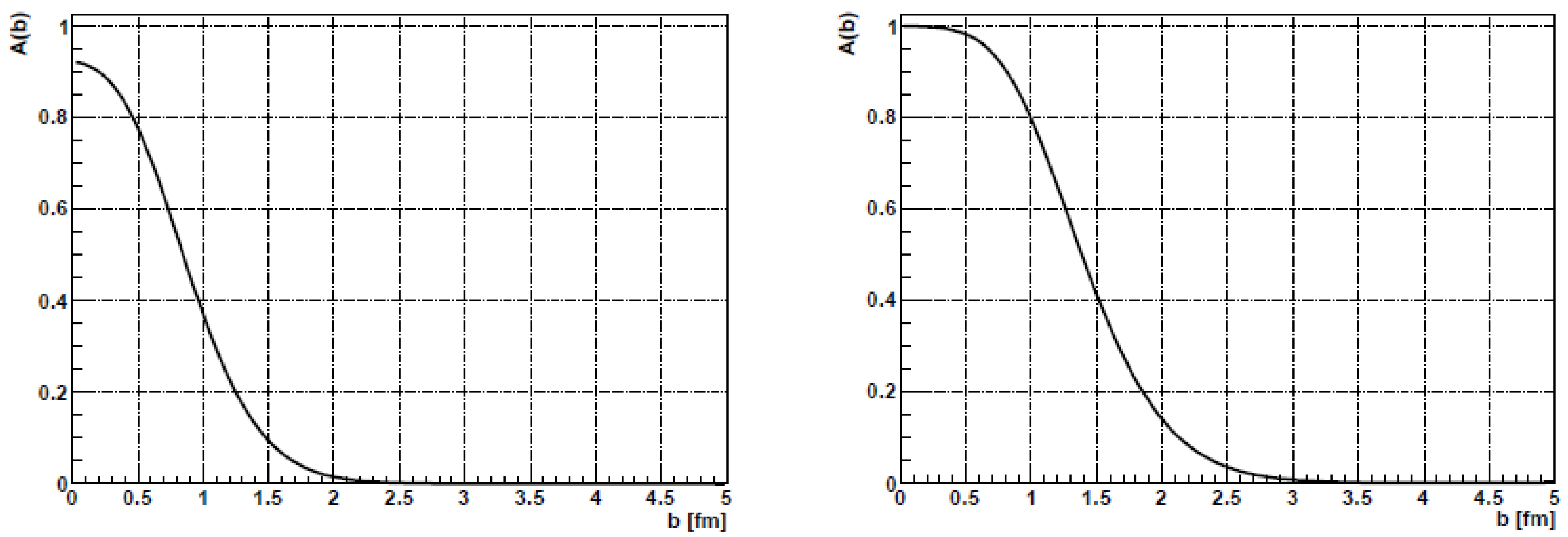
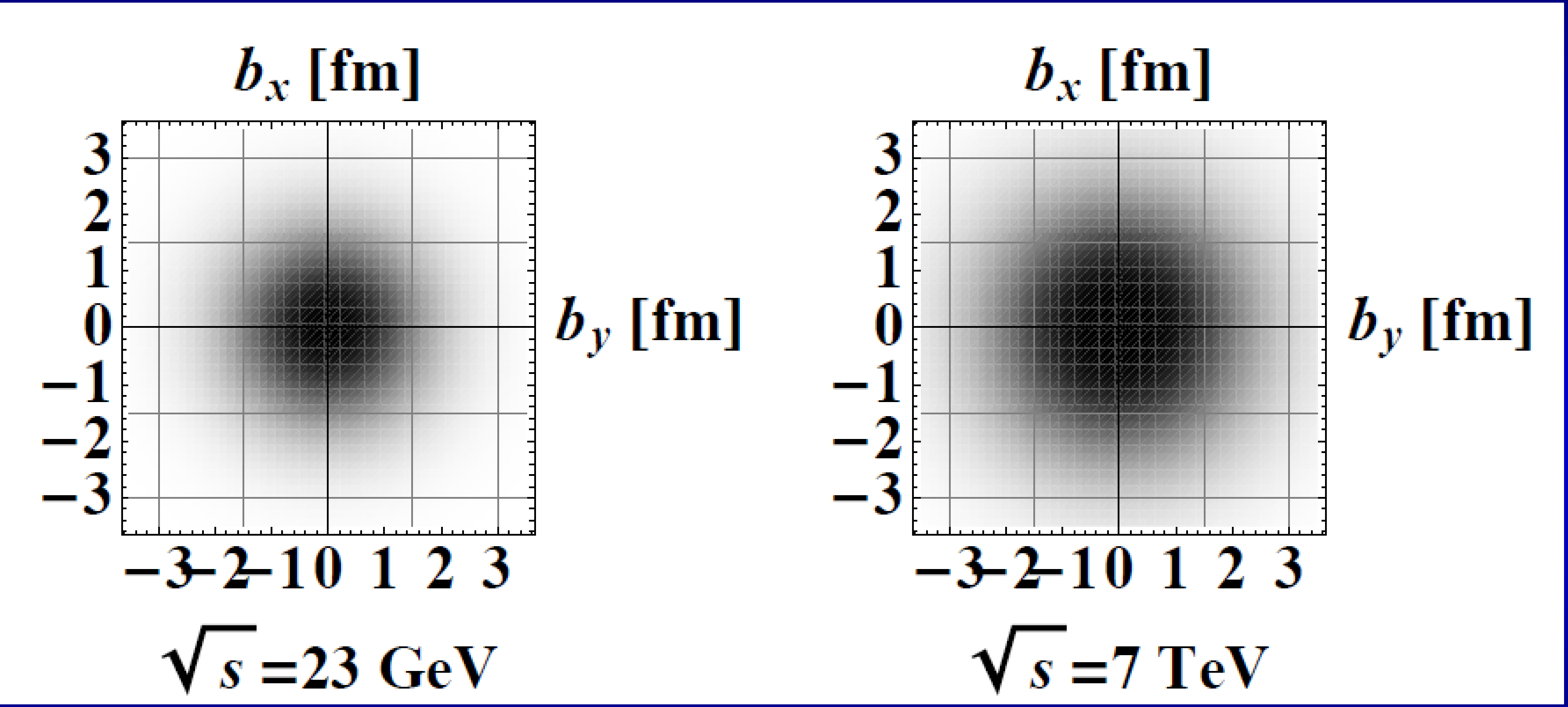


Figure 4: The $A(b) = 1 - |e^{-\Omega(b)}|^2$ shadow profile function. 23.5 GeV (left) and 7 TeV (right).

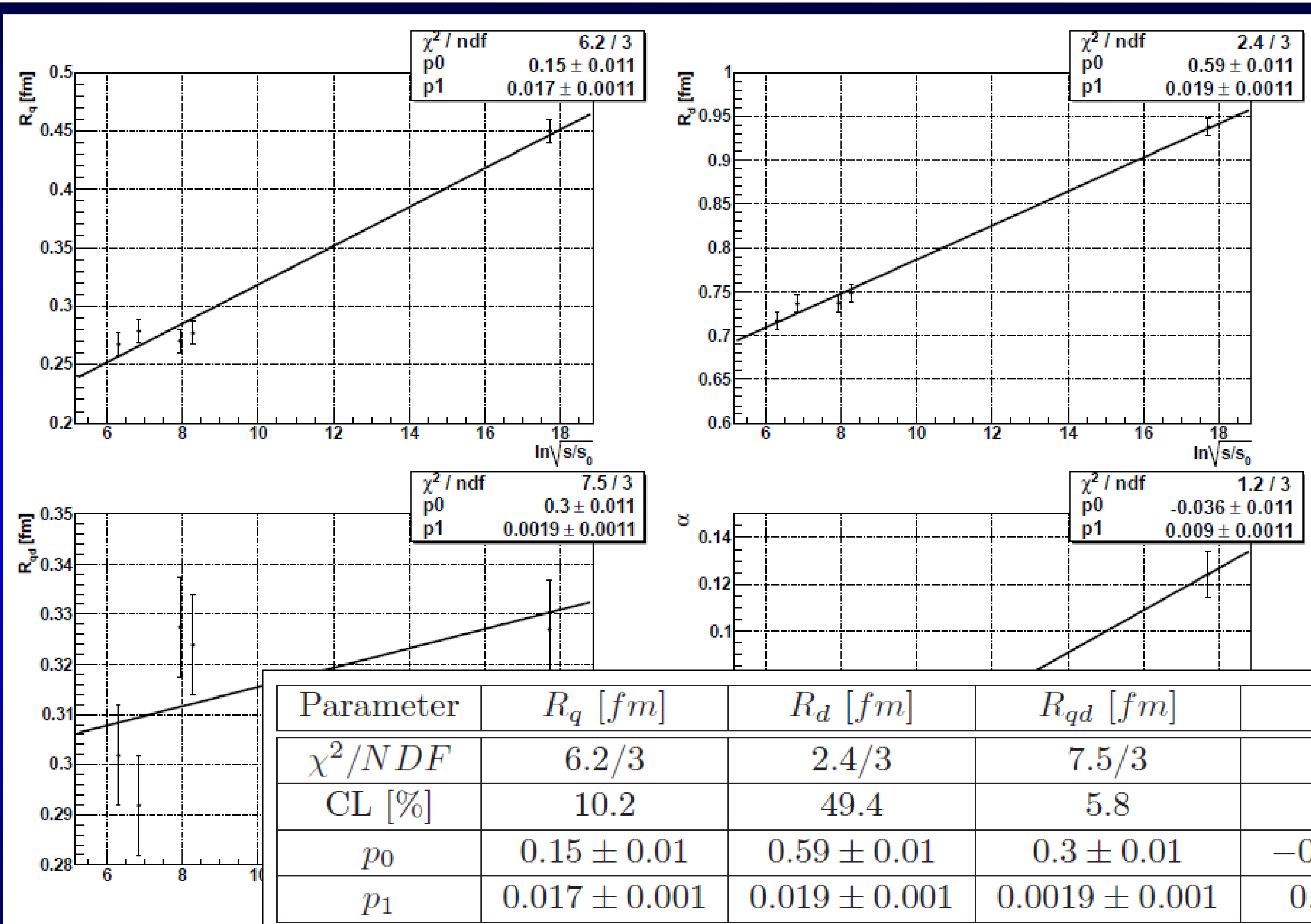
A telítődés jelei 7 TeV-nél: A(b) ~ 1 a kis b tartományban.
~ max kölcsönhatási valószínűség, ha b kicsi.

Képalkotás a szub-femtométer skálán
23 GeVes ISR és 7 TeVes LHC adatokon



8 és 13 TeV-es és későbbi LHC energiákon?

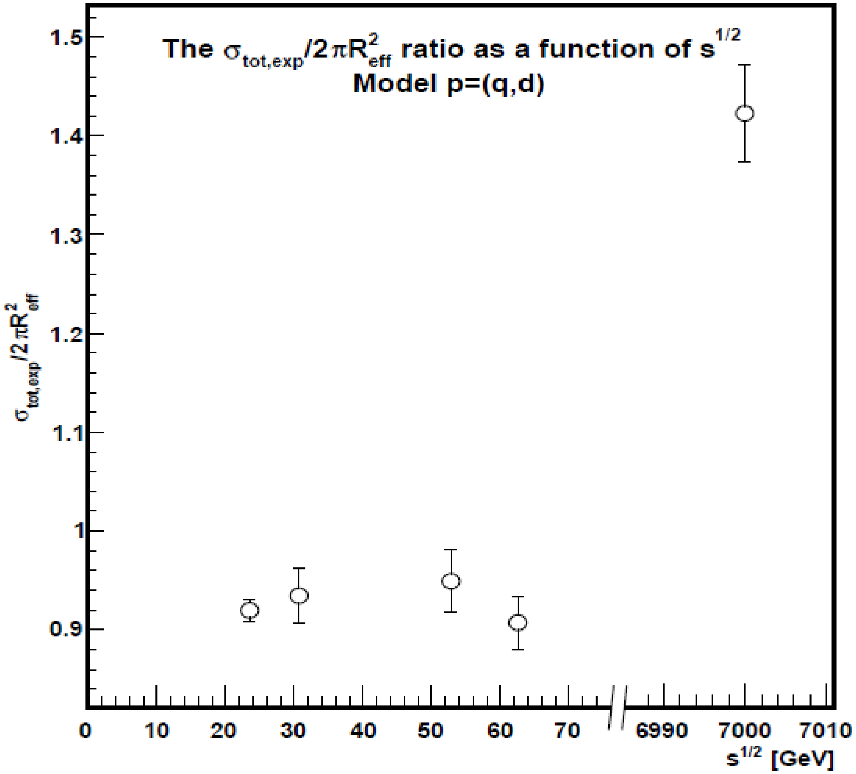
Geometriai skálázás a pp szórásban

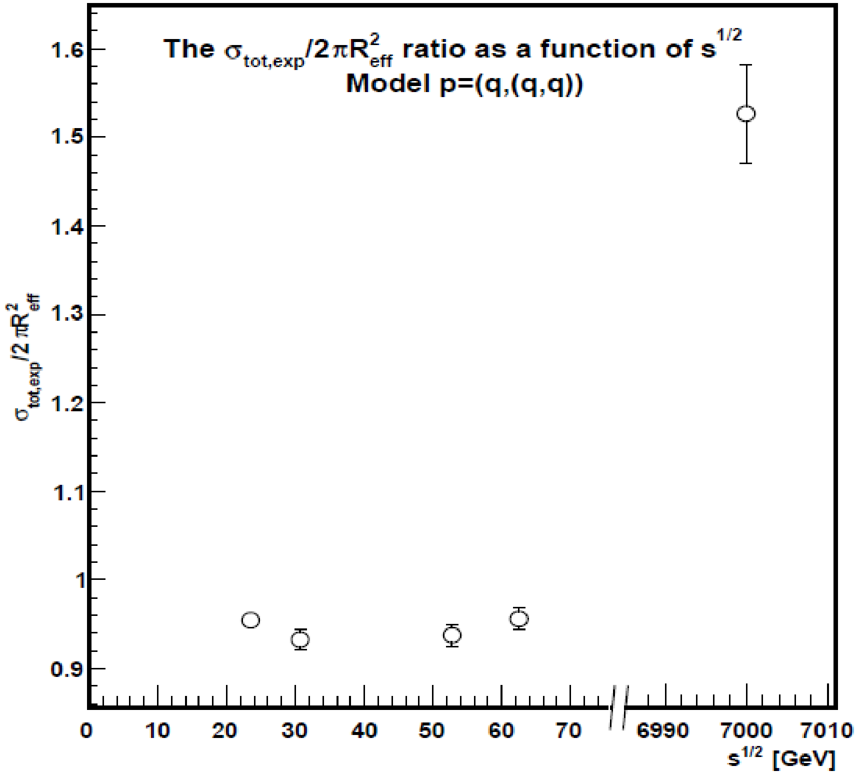


Parameter	$R_q\ [fm]$	$R_d\ [fm]$	$R_{qd}\ [fm]$	α
χ^2/NDF	$6.2/3$	$2.4/3$	$7.5/3$	$1.2/3$
CL [%]	10.2	49.4	5.8	75.3
p_0	0.15 ± 0.01	0.59 ± 0.01	0.3 ± 0.01	-0.036 ± 0.01
p_1	0.017 ± 0.001	0.019 ± 0.001	0.0019 ± 0.001	0.009 ± 0.001

Geometria skálázás: $\{R_q, R_d, R_{qd}, \alpha\} = p_0 + p_1 \ln (s/s_0)$

Mit tudtunk meg eddig?



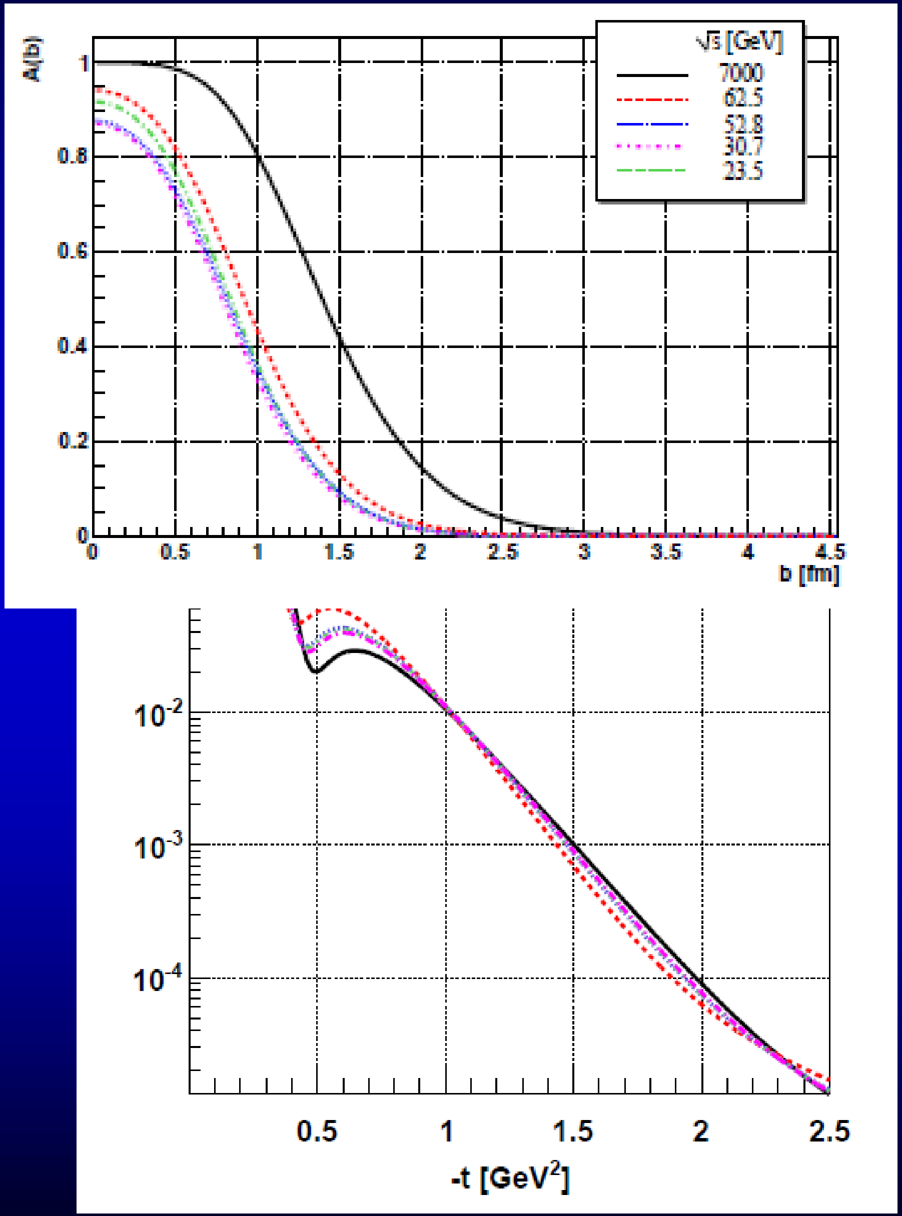


Modell független effektív formula:
jó közelítés, BB, αBB és ReBB modellekre
F. Nemes and T. Cs, arXiv:1204.5617
→ Froissart-Martin korlát rendben (új)!

$$R_{\rm eff} = \sqrt{R_q^2 + R_d^2 + R_{qd}^2}\,,$$

$$\sigma_{total} = 2\pi R_{\rm eff}^2\,.$$

Mit tudtunk meg eddig (2)?



BB: Bialas-Bzdak

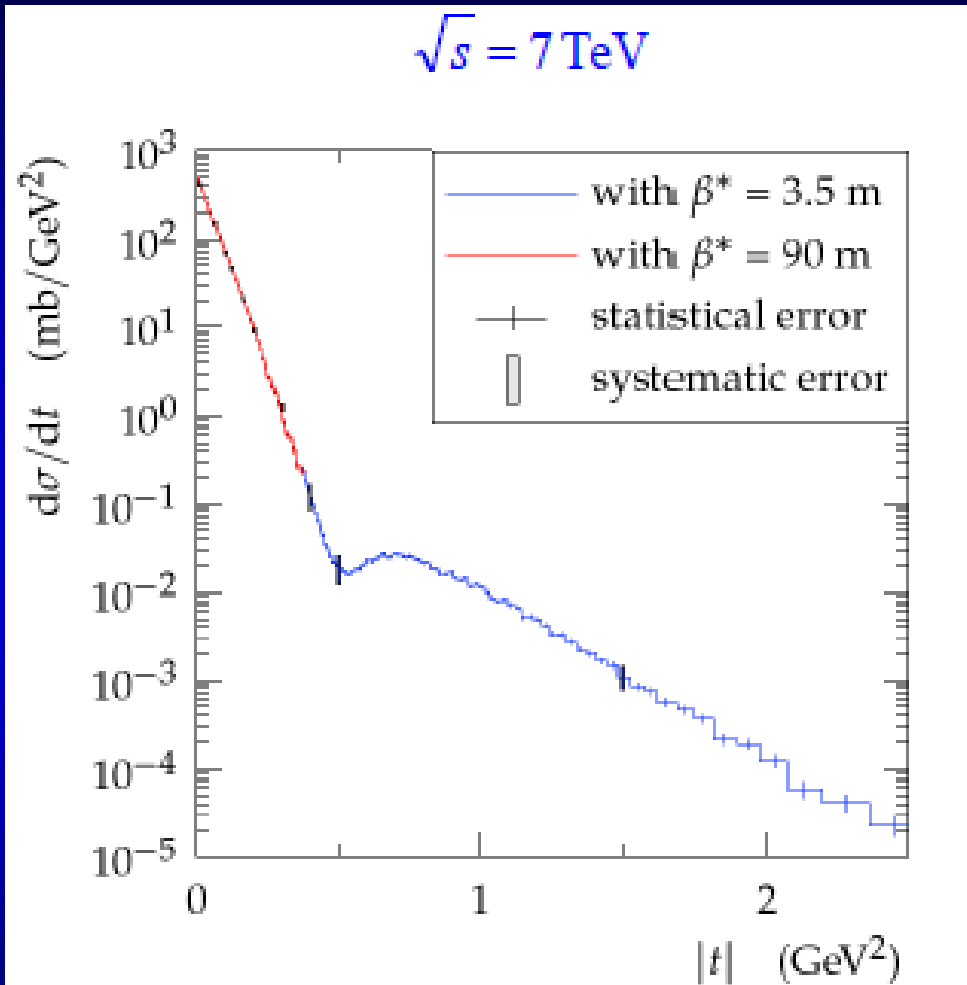
Gauss b-ben:
Pomeron t-ben

ReBB modell:
Pomeron Femtoszkópia
p = (q,d)

ReBB modell:
Froissart-Martin felső korlát
Automatikusan teljesül!

BB → AA modell

Earlier results on elastic scattering



Earlier hints on non-exponential behaviour:

at ISR: 21.5 to 52.8 GeV, change of slope and better fits with exp(-B |t|- C t^2)

at SppS: Change of slope only, at |t|~ 0.14 GeV^2

At Tevatron, non-exponential not seen

earlier LHC data ~ exponential, satisfactory fits with exp(-B |t|). New TOTEM data at low |t|: evidence for non-exponential

Glauber–Velasco: Multiple Diffraction Theory

$$F(t) = i \int_0^\infty J_0\left(b\sqrt{-t}\right)\{1 - \exp\left[-\Omega\left(b\right)\right]\} \, bdb$$

$$\Omega\left(b\right) = \frac{\kappa}{4\pi}\left(1 - i\alpha\right)\int_0^\infty J_0\left(q\,b\right) G_{p,E}^2\left(-t\right)\frac{f(t)}{f(0)} qdq$$

$$\frac{f(t)}{f(0)} = \frac{e^{i\left(b_1|t| + b_2\,t^2\right)}}{\sqrt{1 + a\left|t\right|}}$$

$$G_{p,E}\left(q^2\right) = \sum_{i=1}^{n} \frac{a_i^E \left(m_i^E\right)^2}{\left(m_i^E\right)^2 + q^2}\,,\ \sum_{i=1}^{n} a_i^E = 1\,,\ G_{p.E}(0) = 1$$

$$d\sigma_{el}/d\left|t\right| = \pi\left|F(t)\right|^2$$

a_i^E	$(m_i^E)^2$ (fm^{-2})
0.219	3.53
1.371	15.02
-0.634	44.08
0.044	154.20

F(t): f. sc. amplitude
Ω(b): opacity, complex

f(t): cluster averaged parton-parton scattering amplitude

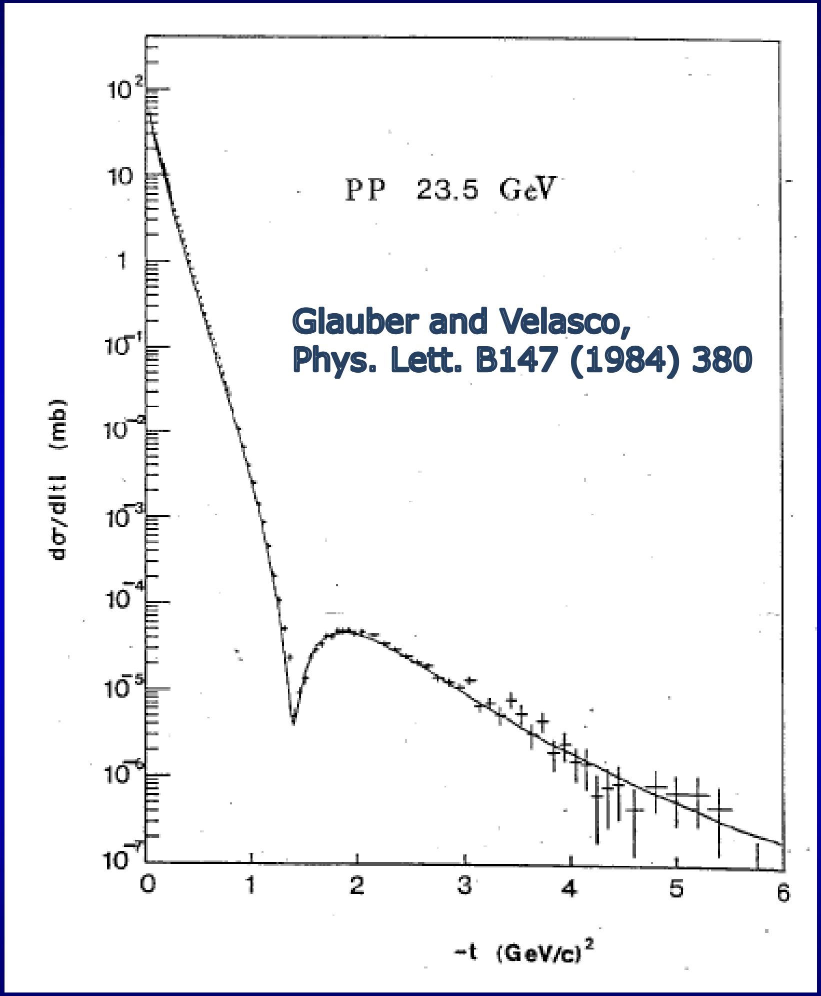
-t = q^2: momentum transfer
b: impact parameter

dσ/dt: diff. cross-section elastic pp scattering

R.J. Glauber and J.Velasco
Phys. Lett. B147 (1987) 380

BSWW EM form factors G_E

Diffraction in pp @ ISR,
Glauber and Velasco, PLB147 (1984) 380



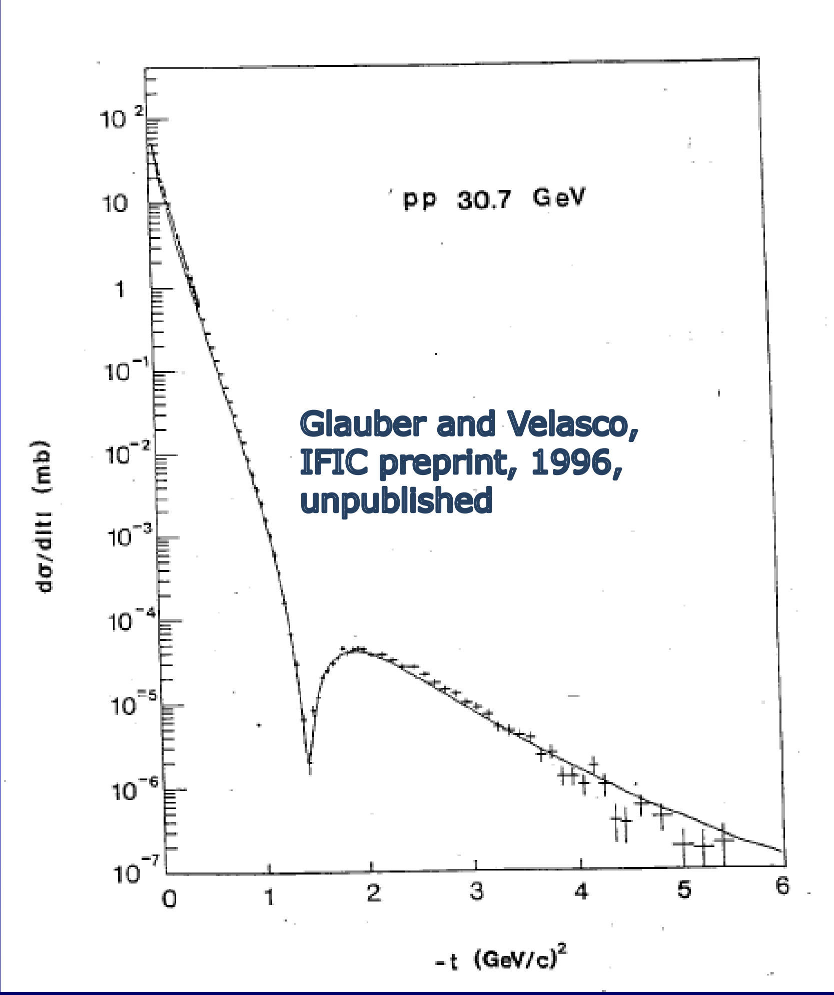
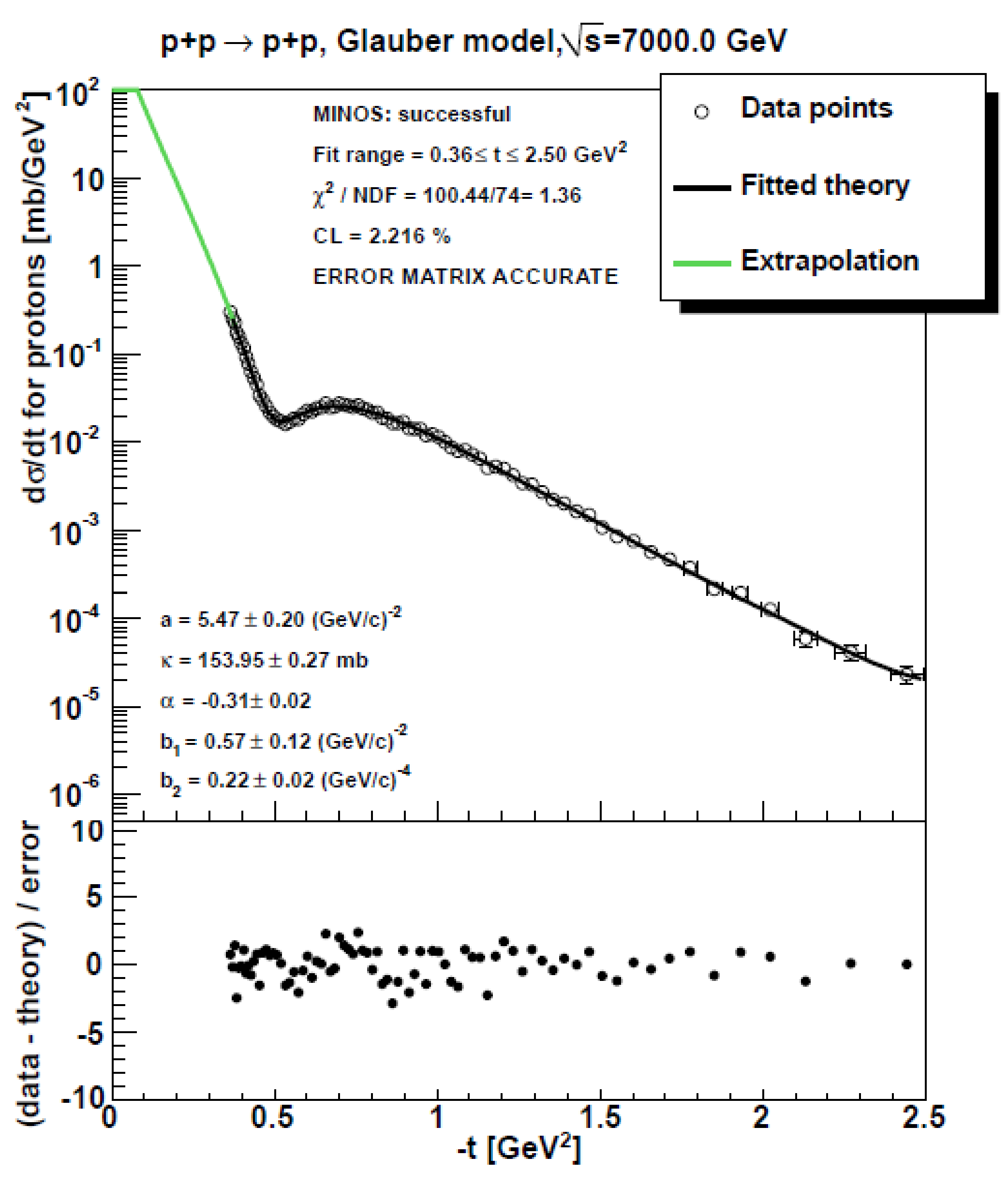


Illustration: elastic pp at the ISR energy range
13.7 – 62.7 GeV well described by Glauber-Velasco

First results @ Low-X 2013: GV works for dσ/dt dip



Glauber-Velasco (GV)
(original)

describes dσ/dt data
Both at ISR and
TOTEM@LHC
in the dip region

arxiv:1311.2308

Note: at low-t
GV is ~ exponential

Really?
Lower energies?

GV predicted non-exponential dσ/dt in 1984

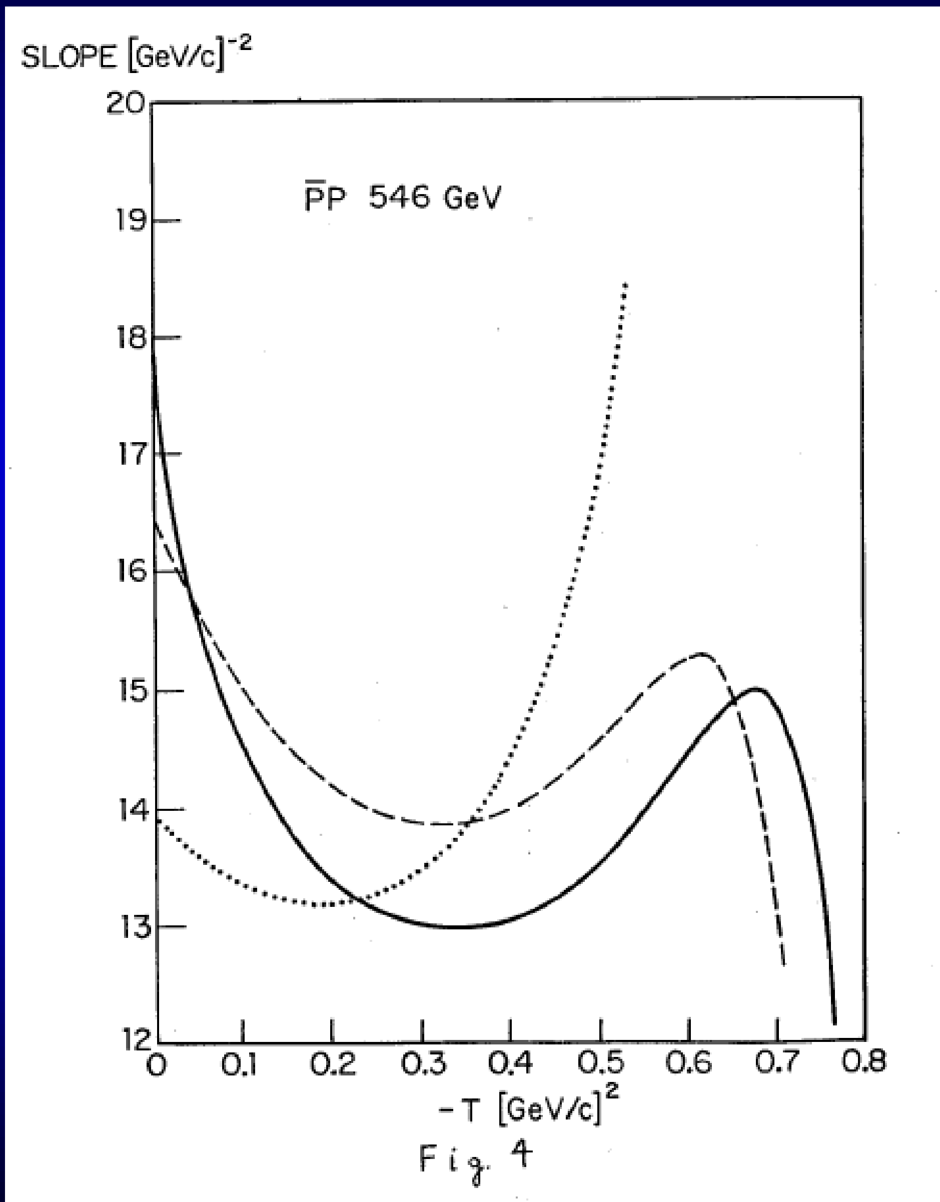


Figure 4 from
Glauber-Velasco
PLB 147 (1984) 380
Slope is not quite
Exponential:
a non-Gaussian behaviour

Quark-Diquark Models
(Real Extended Bialas-Bzdak, 2015)
Non-exponential dσ/dt:
a non-Gaussian behaviour of
A(b) shadow profile function

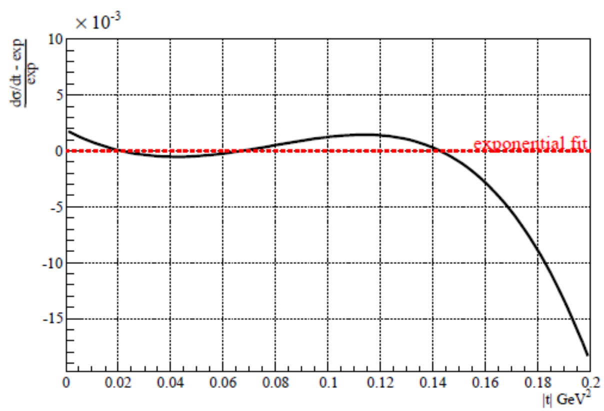
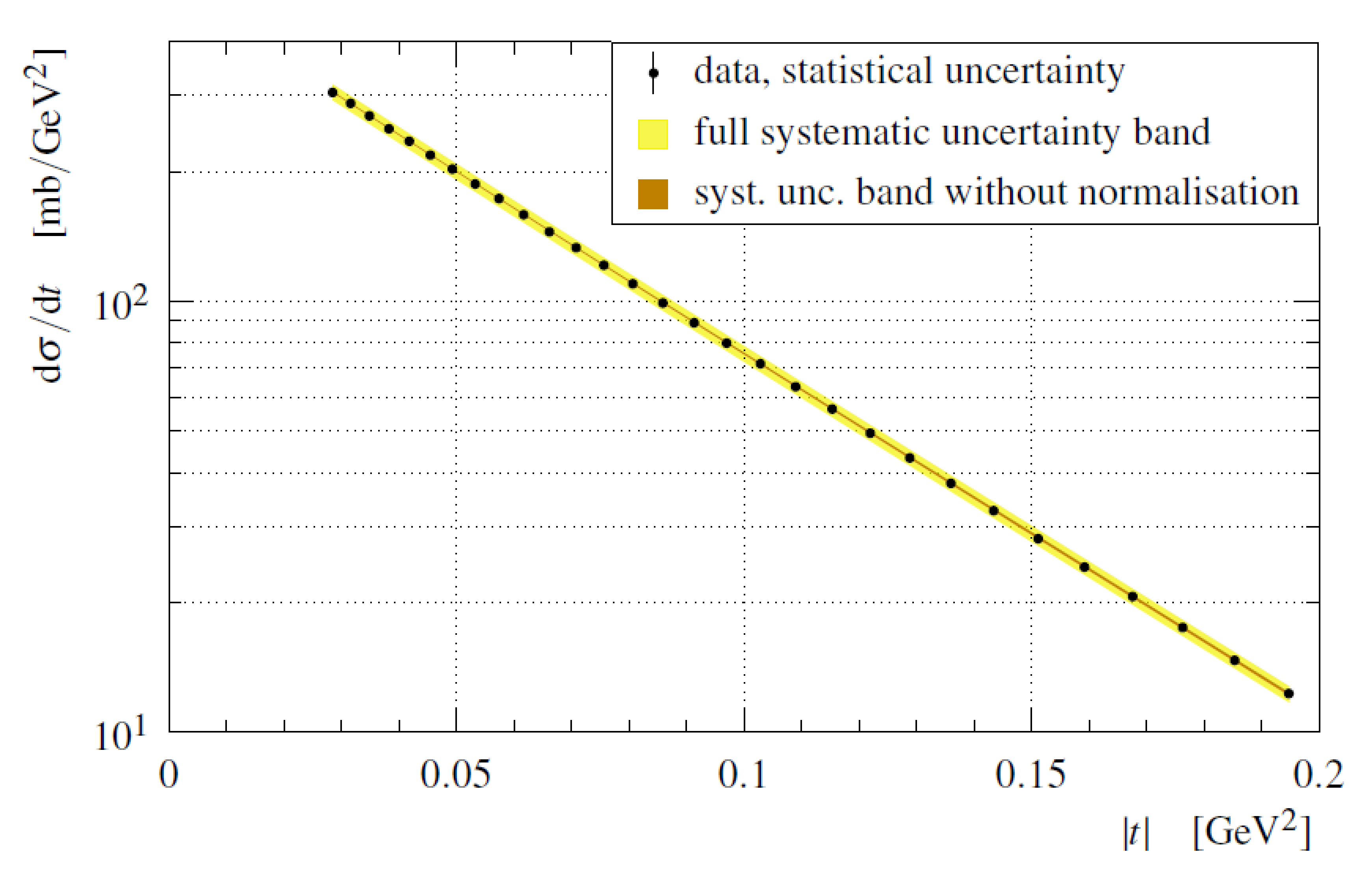


Fig. 5. The ReBB model, fitted in the $0.0 \le |t| \le 0.36$ GeV^2 range, with respect to the exponential fit of Eq. (33). In the plot only the $0.0 \le |t| \le 0.2$ GeV^2 range is shown. The curve indicates a significant deviation from the simple exponential at low $|t|$ values.

TOTEM dσ/dt @ 8 TeV



$t = -p^2\,\theta_*^2$; „optimized binning”; almost exponential but if one looks in detail, NOT

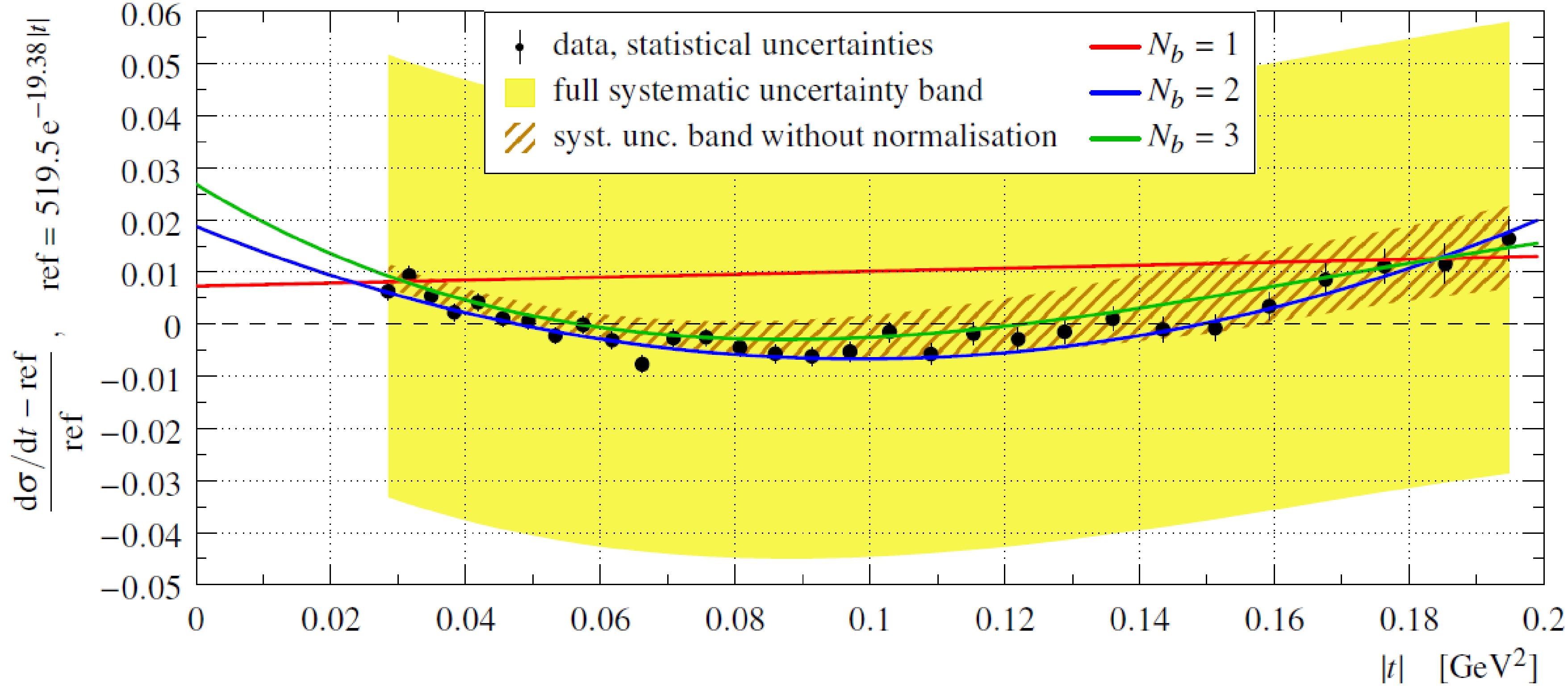
TOTEM news

arxiv:1503.08111, Nucl. Phys. B899 (2015) 527

non-exponential behaviour at low-t

Table 4: Fit quality measures for fits in Figure 11.

N_b	χ^2/ndf	p-value	significance
1	117.5/28 = 4.20	$6.1 \cdot 10^{-13}$	$7.2\,\sigma$
2	29.3/27 = 1.09	0.35	$0.94\,\sigma$
3	25.5/26 = 0.98	0.49	$0.69\,\sigma$



TOTEM Cross-check: modified binning

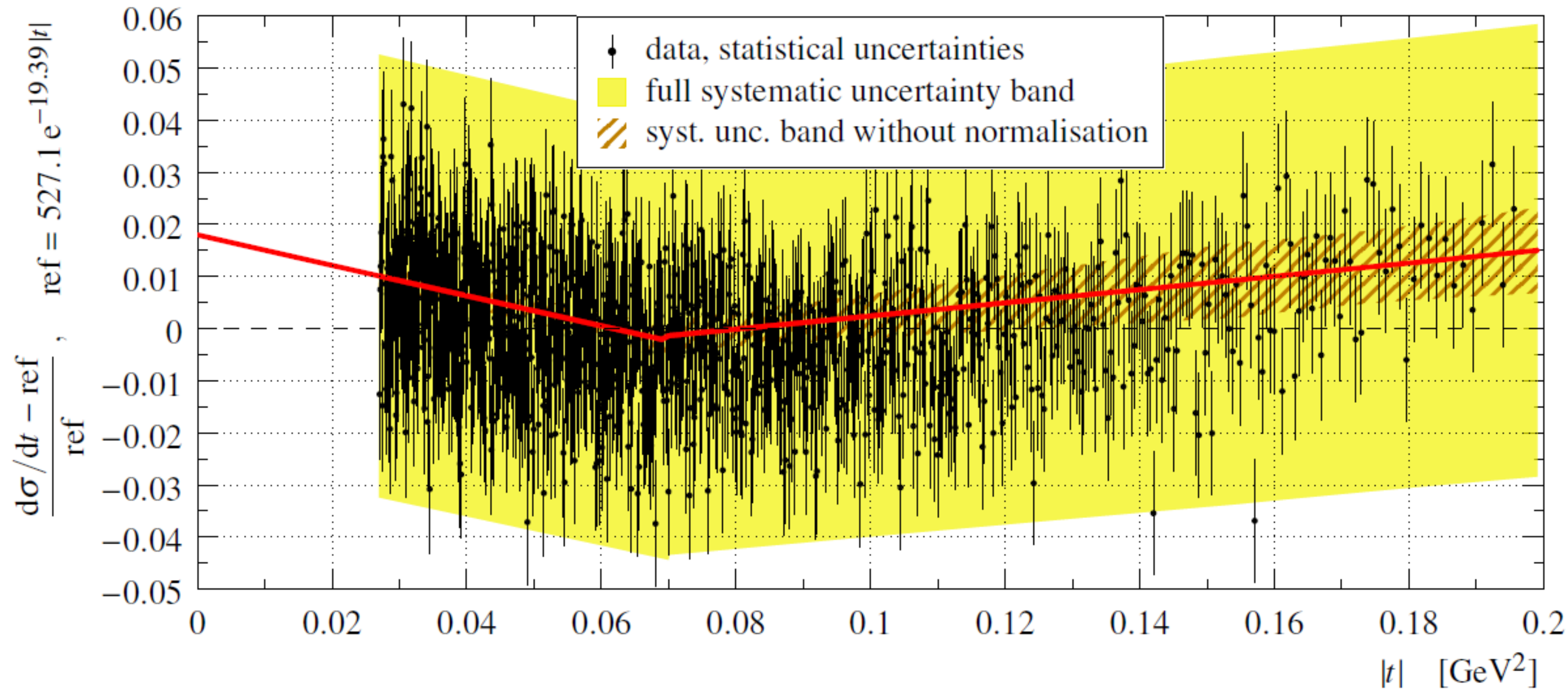


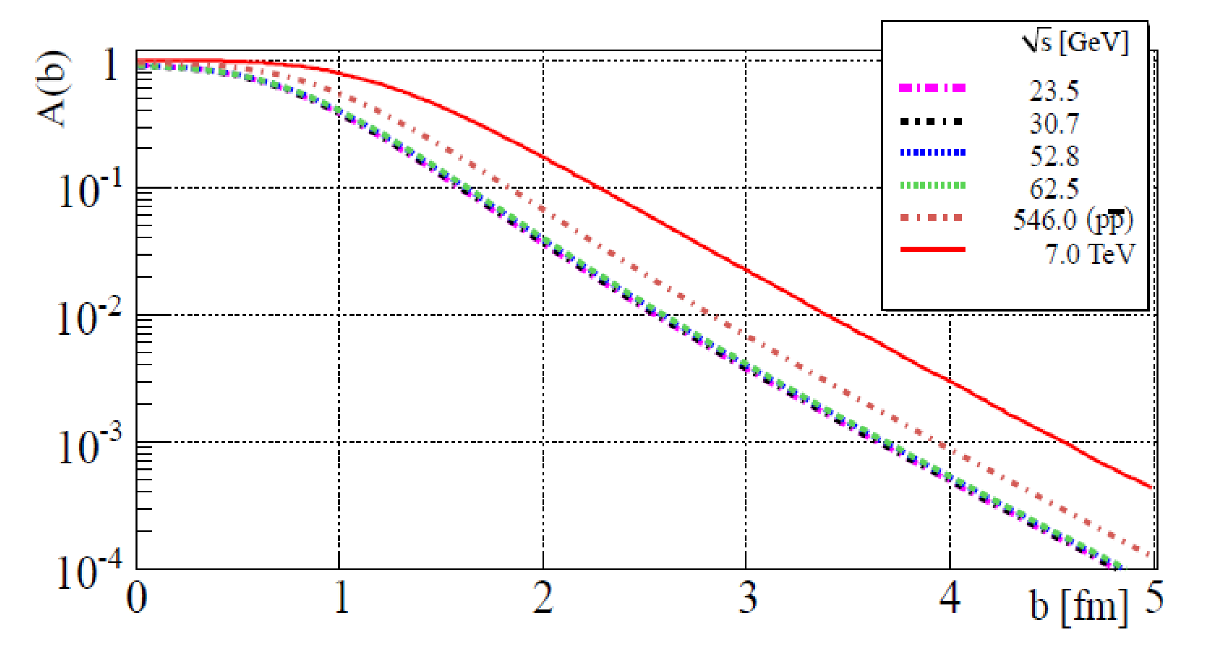
Figure 12: Differential cross-section using the "per-mille" binning and plotted as relative difference from the reference exponential (see vertical axis). The black dots represent data points with statistical uncertainty bars. The red line shows pure exponential fits in regions below and above $|t| = 0.07\,\mathrm{GeV}^2$, see Eq. (19). The yellow band corresponds to the full systematic uncertainty, the brown-hatched one includes all systematic contributions except the normalisation. Both bands are centred around the fit curve.

$$\frac{d\sigma}{dt}(t) = \begin{cases} a_1\, e^{b_1|t|} & |t| < 0.07\,\mathrm{GeV}^2 \\ a_2\, e^{b_2|t|} & |t| > 0.07\,\mathrm{GeV}^2 \end{cases}$$

$$\chi_p^2 = \Delta_p^T V_p^{-1} \Delta_p\,, \qquad \Delta_p = \begin{pmatrix} a_1 - a_2 \\ b_1 - b_2 \end{pmatrix}$$

Simple exp fits excluded. Different binnings show the same effect. Here 7.8 σ significance.

Saturation from shadow profiles



at 7 TeV
proton becomes

Blacker, but
NOT Edgier,
and Larger

BEL ➡ BneL effect

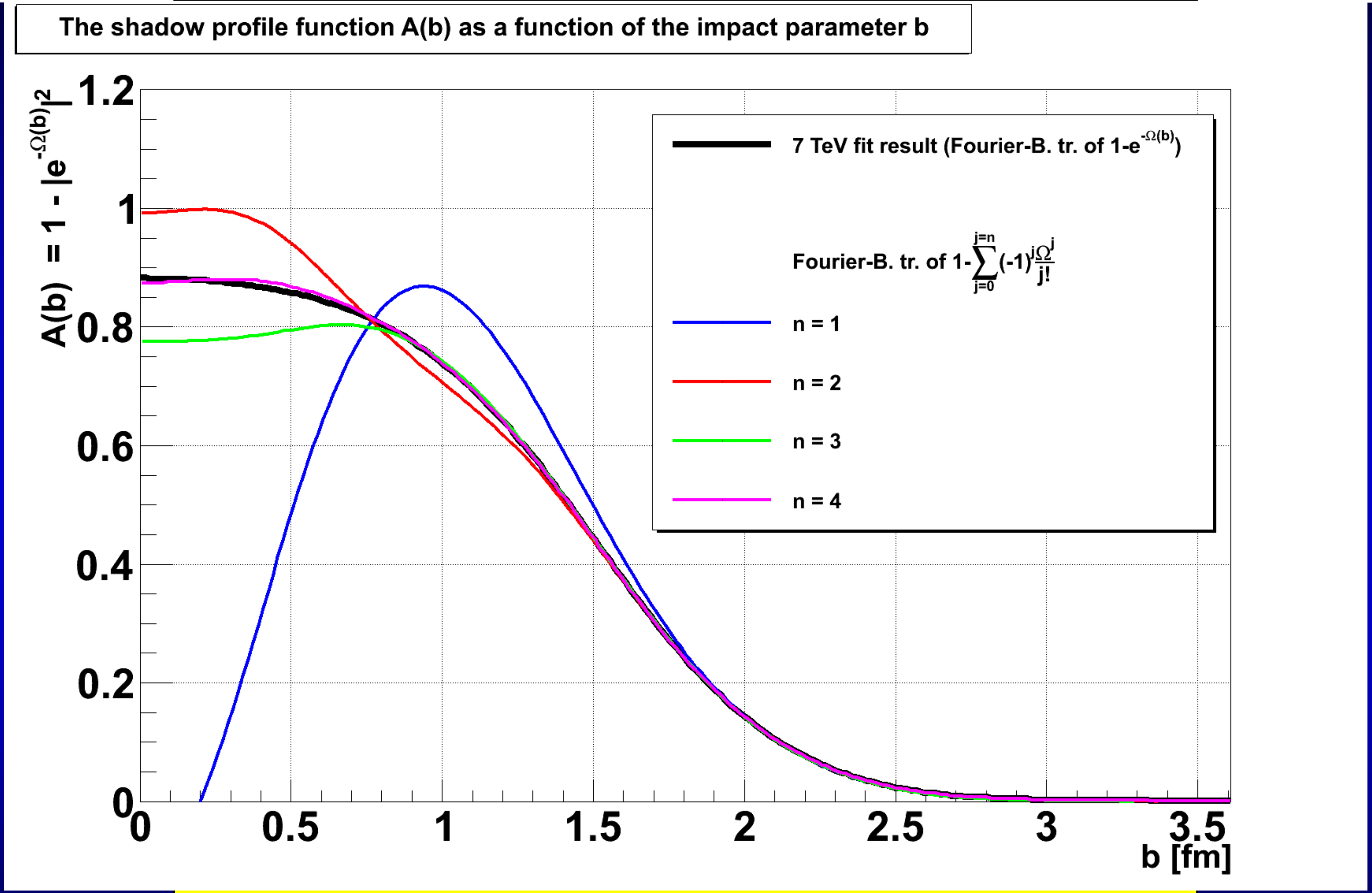
$$A(b) = 1 - |e^{-\Omega(b)}|^2$$

ISR and SppS:
R.J. Glauber and J.Velasco
Phys. Lett. B147 (1987) 380
b_1, b_2 fixed

apparent saturation:
proton is ~ black at LHC
up to
r ~ 0.7 fm

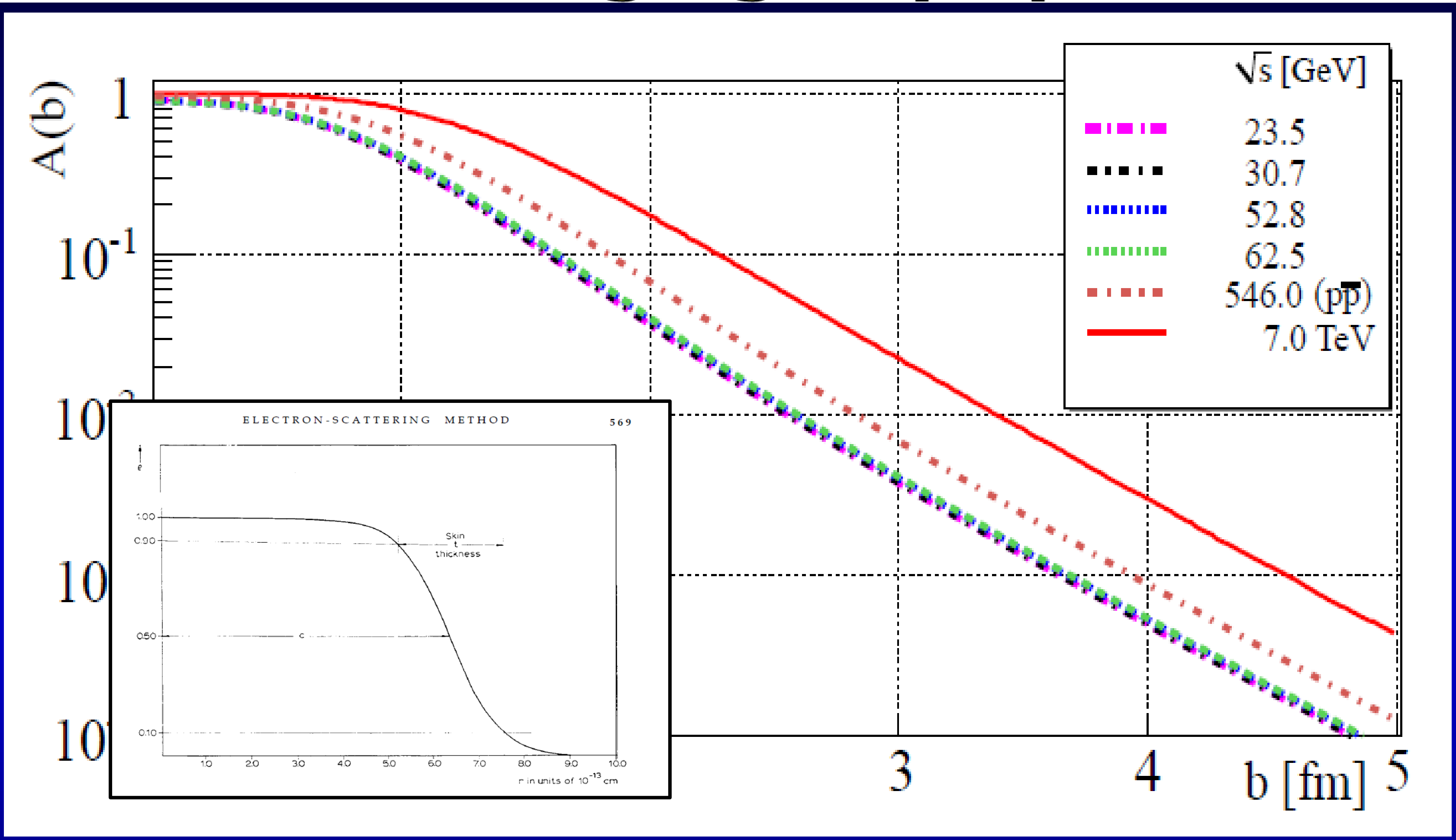
see also Lipari and Lusignoli,
arXiv:1305.7216

Summary



Grey at r~ 1 fm,
black up to r~0.7 fm

Shadow imaging in p+p at LHC



The BneL effect. Non-Gaussian behaviour.
Glauber-Velasco yields results similar to Bialas-Bzdak.
→ Model independent analysis?

Model independent analysis of nearly Levy correlations

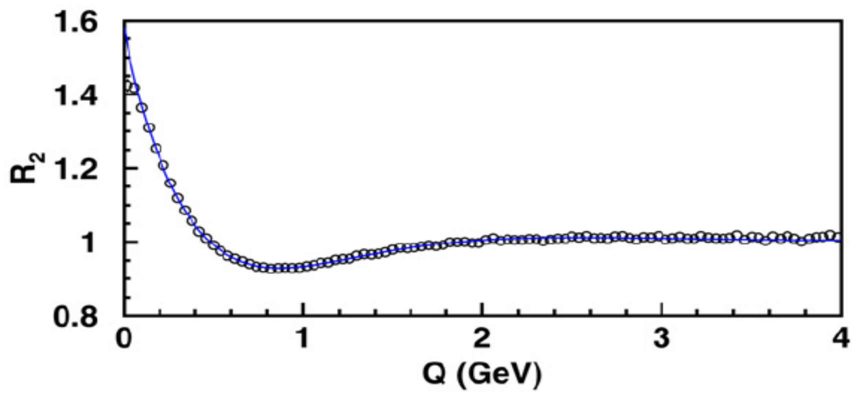


Fig. 1. The Bose–Einstein correlation function R_2 for events generated by PYTHIA. The curve corresponds to a fit of the one-sided Lévy parametrization, Eq. (13).

T. Csörgő et al. / Physics Letters B 663 (2008) 214–216

T. Novák
KRF, Wigner RCP

T. Csörgő
KRF, Wigner RCP

H.C. Eggers and M.B. De Kock
University of Stellenbosh

Model-independent shape analysis of correlations:

- General introduction
- Edgeworth,
- Laguerre,
- Levy expansions

Summary

MODEL - INDEPENDENT SHAPE ANALYIS I.

experimental properties:

i) The correlation function tends to a constant for large values of the relative momentum Q.

ii) The correlation function has a non-trivial structure at a certain value of its argument.

The location of the non-trivial structure in the correlation function is assumed for simplicity to be close to $Q = 0$.

Model-independent but experimentally testable:

- $w(t)$ measure in an abstract H-space
- approximate form of the correlations
- t: dimensionless scale variable

$$\int dt w(t) h_n(t) h_m(t) = \delta_{n,m},$$

$$f(t) = \sum_{n=0}^{\infty} f_n\, h_n(t),$$

$$f_n = \int dt w(t)\, f(t)\, h_n(t).$$

e.g. $t = Q_I R_I$.

MODEL - INDEPENDENT SHAPE ANALYIS

$$C_2(\mathbf{k}_1, \mathbf{k}_2) = \frac{N_2(\mathbf{k}_1, \mathbf{k}_2)}{N_1(\mathbf{k}_1)\, N_1(\mathbf{k}_2)},$$

$$R_2(\mathbf{k}_1, \mathbf{k}_2) = C_2(\mathbf{k}_1, \mathbf{k}_2) - 1.$$

Let us assume, that the function $g(t) = R_2(t)/w(t)$ is also an element of the Hilbert space H. This is possible, if

$$\int dt\, w(t) g^2(t) = \int dt\, \left[R_2^2(t)/w(t)\right] < \infty, \qquad (6)$$

Then the function *g* can be expanded as

$$g(t) = \sum_{n=0}^{\infty} g_n h_n(t),$$

$$g_n = \int dt\, R_2(t) h_n(t).$$

From the completeness of the Hilbert space and from the assumption that *g*(*t*) is in the Hilbert space:

$$R_2(t) = w(t) \sum_{n=0}^{\infty} g_n h_n(t).$$

T. Csörgő and S. Hegyi, hep-ph/9912220, T.

MODEL - INDEPENDENT SHAPE ANALYIS III.

$$C_2(\mathbf{k}_1, \mathbf{k}_2) = \frac{N_2(\mathbf{k}_1, \mathbf{k}_2)}{N_1(\mathbf{k}_1)\, N_1(\mathbf{k}_2)},$$

$$C_2(t) = \mathcal{N} \left\{ 1 + \lambda_w\, w(t) \sum_{n=0}^{\infty} g_n h_n(t) \right\}$$

Model-independent AND experimentally testable:

- method for any approximate shape $w(t)$
- the core-halo intercept parameter of the CF is
- coefficients by numerical integration (fits to data)
- condition for applicability: experimentally testabe

$$\lambda_* = \lambda_w \sum_{n=0}^{\infty} g_n h_n(0)$$

$$g_n = \int dt\, R_2(t) h_n(t)$$

$$\int dt \left[R_2^2(t) / w(t) \right] < \infty$$

EDGEWORTH EXPANSION: ~ GAUSSIAN

$$t = \sqrt{2} Q R_E,$$

$$w(t) = \exp(-t^2/2),$$

$$\int_{-\infty}^{\infty} dt \, \exp(-t^2/2) H_n(t) H_m(t) \propto \delta_{n,m},$$

$$H_n(t) = \exp(t^2/2) \left(-\frac{d}{dt} \right)^n \exp(-t^2/2).$$

$$H_1(t) = t,$$
$$H_2(t) = t^2 - 1,$$
$$H_3(t) = t^3 - 3t,$$
$$H_4(t) = t^4 - 6t^2 + 3, \; \ldots$$

$$C_2(Q) = \mathcal{N} \left\{ 1 + \lambda_E \exp(-Q^2 R_E^2) \times \left[1 + \frac{\kappa_3}{3!} H_3(\sqrt{2} Q R_E) + \frac{\kappa_4}{4!} H_4(\sqrt{2} Q R_E) + \ldots \right] \right\}.$$

3d generalization straightforward but not discussed

- Applied by NA22, L3, STAR, PHENIX, ALICE, CMS (LHCb?)

LAGUERRE EXPANSIONS: ~ EXPONENTIAL

Model-independent but experimentally tested:

- $w(t)$ exponential
- t: dimensionless
- Laguerre polynomials

$$t = QR_L,$$
$$w(t) = \exp(-t)$$
$$\int_0^\infty dt\, \exp(-t) L_n(t) L_m(t) \propto \delta_{n,m},$$

$$L_n(t) = \exp(t) \frac{d^n}{dt^n} (-t)^n \exp(-t).$$

$$L_0(t) = 1,$$
$$L_1(t) = t - 1,$$

$$C_2(Q) = \mathcal{N} \left\{ 1 + \lambda_L \exp(-QR_L) \left[1 + c_1 L_1(QR_L) + \frac{c_2}{2!} L_2(QR_L) + \dots \right] \right\}$$

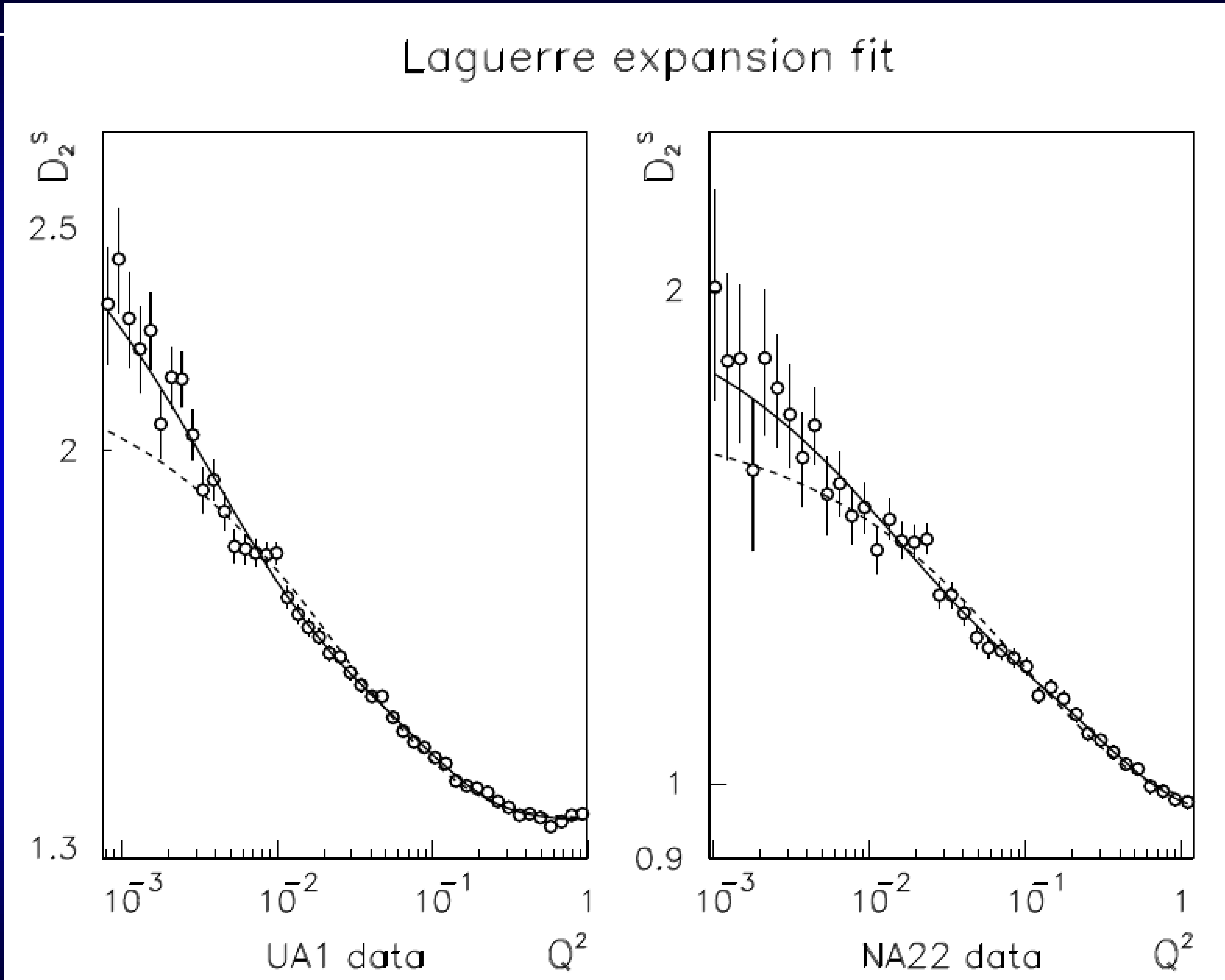
First successful tests

- NA22, UA1 data
- convergence criteria satisfied
- intercept parameter ~ 1

$$\int_0^\infty dt\, R_2^2(t) \exp(+t) < \infty,$$

$$\lambda_* = \lambda_L [1 - c_1 + c_2 - \dots],$$
$$\delta^2 \lambda_* = \delta^2 \lambda_L \left[1 + c_1^2 + c_2^2 + \dots \right] + \lambda_L^2 \left[\delta^2 c_1 + \delta^2 c_2 + \dots \right]$$

LAGUERRE EXPANSIONS: ~ superEXPONENTIAL



T. Csörgő and S. Hegyi, hep-ph/9912220, T. Csörgő, hep-ph/001233

MINIMAL MODEL ASSUMPTION: LEVY

experimental conditions:

(i) The correlation function tends to a constant for large values of the relative momentum Q.

(ii) The correlation function deviates from its asymptotic, large Q value in a certain domain of its argument.

(iii) The two-particle correlation function is related to a Fourier transformed space-time distribution of the source.

Model-independent but:

- **Assumes that Coulomb can be corrected**
- **No assumptions about analyticity yet**
- **For simplicity, consider 1d case first**
- **For simplicity, consider factorizable *x k***
- **Normalizations :**
  - density
  - multiplicity
  - single-particle spectra

$$C_2(\mathbf{k}_1, \mathbf{k}_2) = \frac{N_2(\mathbf{k}_1, \mathbf{k}_2)}{N_1(\mathbf{k}_1)\, N_1(\mathbf{k}_2)}$$

$$S(x, k) = f(x)\, g(k)$$

$$\int \mathrm{d}x\, f(x) = 1, \qquad \int \mathrm{d}k\, g(k) = \langle n \rangle,$$

$$N_1(k) = \int \mathrm{d}x\, S(x, k) = g(k).$$

T. Cs, S. Hegyi, W.A. Zajc, EPJ C36, 67 (2004)

MINIMAL MODEL ASSUMPTION: LEVY

Model-independent but:

- **not assumes analyticity**
- **C_2 measures a modulus squared Fourier-transform vs relative momentum**
- **Correlations non-Gaussian**
- **Radius not a variance**
- **$0 < \alpha \leq 2$**

$$C_2(k_1, k_2) = 1 + |\tilde{f}(q_{12})|^2,$$

$$\tilde{f}(q_{12}) = \int \mathrm{d}x \, \exp(\mathrm{i} q_{12} x) \, f(x),$$

$$C(q;\alpha) = 1 + \lambda \exp\left(-|qR|^{\alpha}\right).$$

T. Cs, S. Hegyi, W.A. Zaic, EPJ C36, 67 (2004)

UNIVARIATE LEVY EXAMPLES

Include some well known cases:

- **α = 2**
  - Gaussian source, Gaussian C_2

$$f(x) = \frac{1}{(2\pi R^2)^{1/2}} \exp\left[-\frac{(x-x_0)^2}{2R^2}\right]$$

$$C(q) = 1 + \exp\left(-q^2 R^2\right)$$

- **α = 1**
  - Lorentzian source, exponential C_2

$$f(x) = \frac{1}{\pi} \frac{R}{R^2 + (x-x_0)^2},$$

$$C(q) = 1 + \exp\left(-|q\,R|\right).$$

- **asymmetric Levy:**
  - asymmetric support
  - Streched exponential

$$f(x) = \sqrt{\frac{R}{8\pi}} \frac{1}{(x-x_0)^{3/2}} \exp\left(-\frac{R}{8(x-x_0)}\right)$$

$$x_0 < x < \infty,$$

$$C(q) = 1 + \exp\left(-\sqrt{|q\,R|}\right).$$

T. Cs, hep-ph/0001233, T. Cs, S. Hegyi, W.A. Zajc, EPJ C36, 67 (2004)

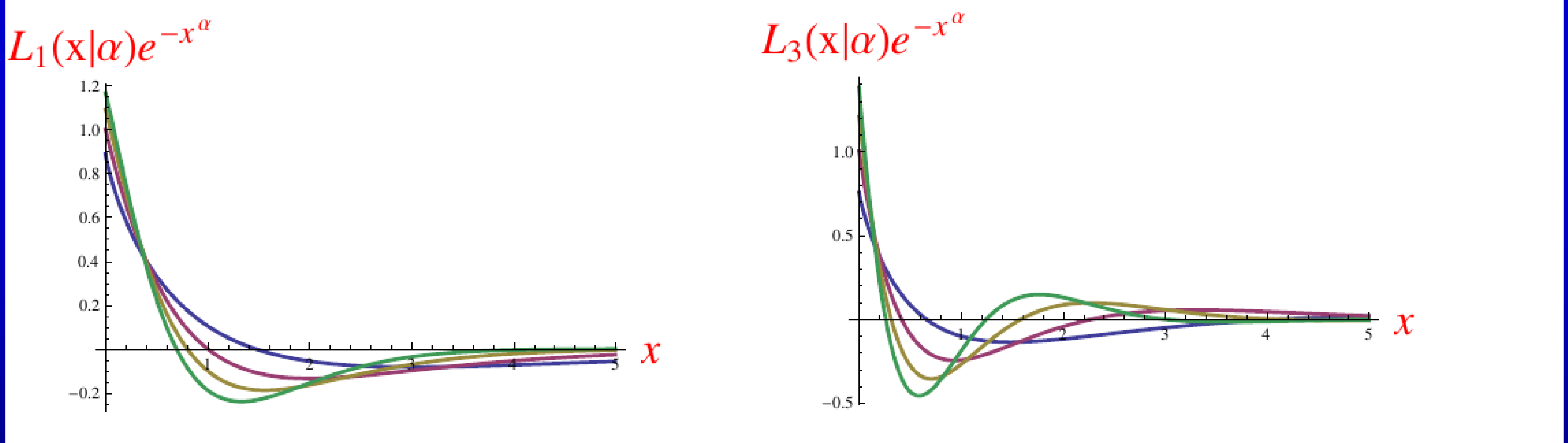
LEVY EXPANSIONS: ~ 1d LEVY

- Levy generalizes exponentials and Gaussians
- ubiquoutous
- How far from a Levy?
- Need new set of polynomials orthonormal to a Levy weight

$$L_0(t\,|\,\alpha) = 1, \qquad L_1(t\,|\,\alpha) = \det\begin{pmatrix} \mu_{0,\alpha} & \mu_{1,\alpha} \\ 1 & t \end{pmatrix}, \qquad L_2(t\,|\,\alpha) = \det\begin{pmatrix} \mu_{0,\alpha} & \mu_{1,\alpha} & \mu_{2,\alpha} \\ \mu_{1,\alpha} & \mu_{2,\alpha} & \mu_{3,\alpha} \\ 1 & t & t^2 \end{pmatrix}$$

$$\mu_{n,\alpha} = \int_0^\infty dt\; t^n \exp(-t^\alpha) = \tfrac{1}{\alpha}\,\Gamma(\tfrac{n+1}{\alpha}).$$

$$\Gamma(z) = \int_0^\infty dt\; t^{z-1} \exp(-t)$$



Lévy polynomials of first and third order times the weight function e^{-x^α} for $\alpha = 0.8, 1.0, 1.2, 1.4$.

1st-order Lévy polynomial	$\gamma\left[1 + \lambda e^{-R^\alpha Q^\alpha}[1 + c_1 L_1(Q\vert\alpha, R)]\right]$
3rd-order Lévy polynomial	$\gamma\left[1 + \lambda e^{-R^\alpha Q^\alpha}[1 + c_1 L_1(Q\vert\alpha, R) + c_3 L_3(Q\vert\alpha, R)]\right]$

M. de Kock, H. C. Eggers, T. Cs. arXiv:1206.1680v1 [nucl-th]

LEVY EXPANSIONS: ~ 1d LEVY

- **In case of $\alpha = 1$ Laguerre is ok**

$$
\begin{aligned}
L_0(t\,|\,\alpha = 1) &= 1, \\
L_1(t\,|\,\alpha = 1) &= t - 1, \\
L_2(t\,|\,\alpha = 1) &= t^2 - 4t + 2.
\end{aligned}
$$

These reduce to the Laguerre expansions and Laguerre polynomials.

T. Csörgő and S. Hegyi, hep-ph/9912220, T.

LEVY EXPANSIONS: ~ 1d LEVY

- **In case of $\alpha = 2$ instead of Edgeworth new formulae for one-sided Gaussian:**

$$
\begin{aligned}
L_0(t\,|\,\alpha = 2) &= 1, \\
L_1(t\,|\,\alpha = 2) &= \frac{1}{2}\left\{\sqrt{\pi} t - 1\right\}, \\
L_2(t\,|\,\alpha = 2) &= \frac{1}{32}\left\{(\pi - 2)t^2 - \sqrt{\pi} t + 2 - \frac{\pi}{2}\right\}.
\end{aligned}
$$

Provides a new expansion around a Gaussian shape that is defined for the non-negative values of *t* only.

T. Csörgő and S. Hegyi, hep-ph/9912220, T.

MULTIVARIATE LEVY EXPANSIONS

$$
\begin{aligned}
L_0(t\,|\,\alpha) &= 1, \\
L_1(t\,|\,\alpha) &= \frac{1}{\alpha}\left\{\Gamma(\frac{1}{\alpha})t - \Gamma(\frac{2}{\alpha})\right\}, \\
L_2(t\,|\,\alpha) &= \frac{1}{\alpha^2}\left\{\left[\Gamma(\frac{1}{\alpha})\Gamma(\frac{3}{\alpha}) - \Gamma^2(\frac{2}{\alpha})\right]t^2 - \left[\Gamma(\frac{1}{\alpha})\Gamma(\frac{4}{\alpha}) - \Gamma(\frac{3}{\alpha})\Gamma(\frac{2}{\alpha})\right]t + \left[\Gamma(\frac{2}{\alpha})\Gamma(\frac{4}{\alpha}) - \Gamma^2(\frac{3}{\alpha})\right]\right\}.
\end{aligned}
$$

1st-order Levy expansion

$$
t = \left(\sum_{i,j=1}^{3} R_{i,j}^2 q_i q_j\right)^{1/2}
$$

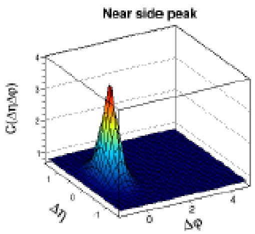
$$
C_2(Q) = N\left\{1 + \lambda \exp\left(-\left(\sum_{i,j=1}^{3} R_{i,j}^2 q_i q_j\right)^{\alpha/2}\right)\left[1 + c_1 \frac{\left(\sum_{i,j=1}^{3} R_{i,j}^2 q_i q_j\right)^{1/2}}{\alpha}\left(\Gamma\left(\frac{1}{\alpha}\right) - \Gamma\left(\frac{2}{\alpha}\right)\right)\right]\right\}
$$

M. de Kock, H. C. Eggers, T. Cs. arXiv:1206.1680v1 [nucl-th]

POSSIBLE APPLICATIONS I

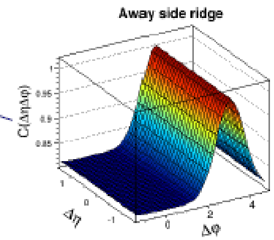
- **Malgorzata Janik's talk at WPCF2014**
- $e_M = \alpha/2$
- **Levy expansion term could be added.**

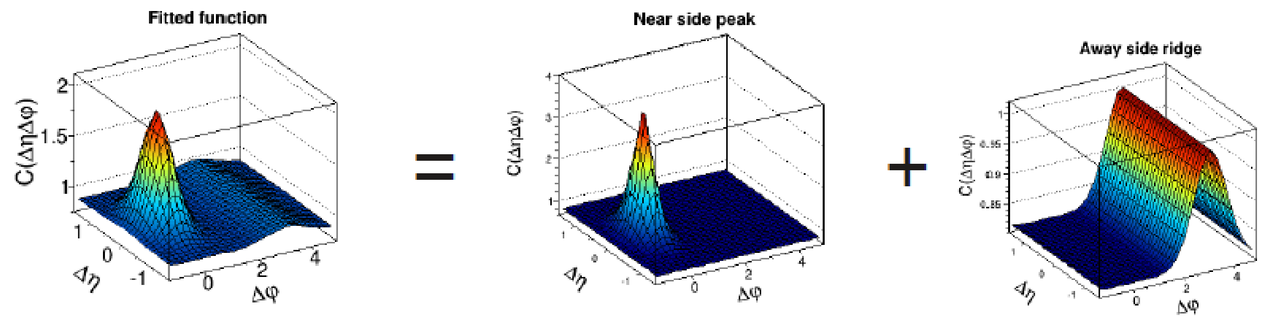
$$C(\Delta\eta,\Delta\varphi) = N$$



$$+ M_M \left\{ \exp\left[-\left(\frac{\Delta\varphi^2}{2\sigma_{M_\varphi}^2} + \frac{\Delta\eta^2}{2\sigma_{M_\eta}^2} \right)^{e_M} \right] + \exp\left[-\left(\frac{\Delta(\varphi-2\pi)^2}{2\sigma_{M_\varphi}^2} + \frac{\Delta\eta^2}{2\sigma_{M_\eta}^2} \right)^{e_M} \right] \right\}$$

$$+ M_A \left\{ \exp\left[-\left(\frac{\Delta(\varphi-\pi)^2}{2\sigma_{A_\varphi}^2} \right) \right] + \exp\left[-\left(\frac{\Delta(\varphi+\pi)^2}{2\sigma_{A_\varphi}^2} \right) \right] \right\}$$





SUMMARY AND CONCLUSIONS

Several model-independent methods:

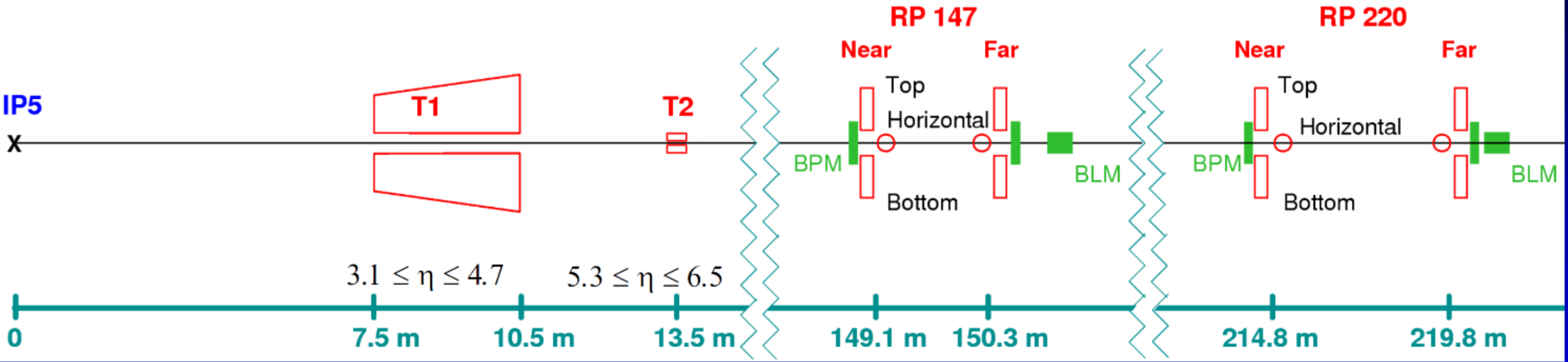
- **Based on matching an abstract measure in *H* to the approximate shape of data**
- **Gaussian: Edgeworth expansions**
- **Exponential: Laguerre expansions**
- **Levy ($0 < \alpha \leq 2$): Levy expansions**
- **In case of alpha = 1 Laguerre ok**
- **In case of alpha = 2 new formulae for Gaussian**
- **New directions: multivariate Levy expansions**

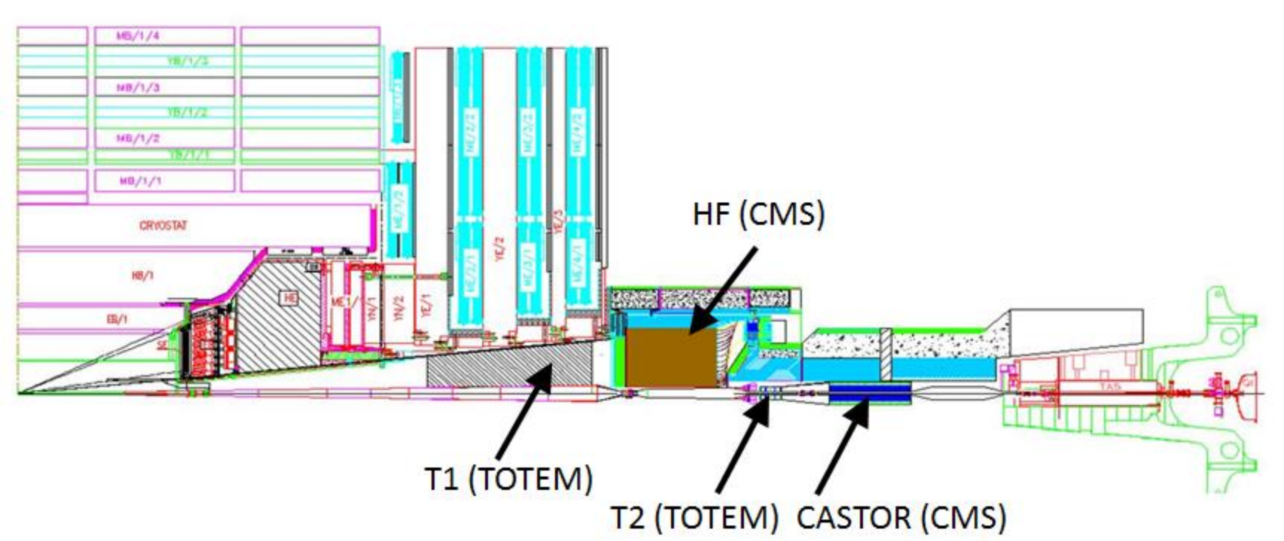
Köszönöm a figyelmet!

Kérdések?

Backup slides - TOTEM

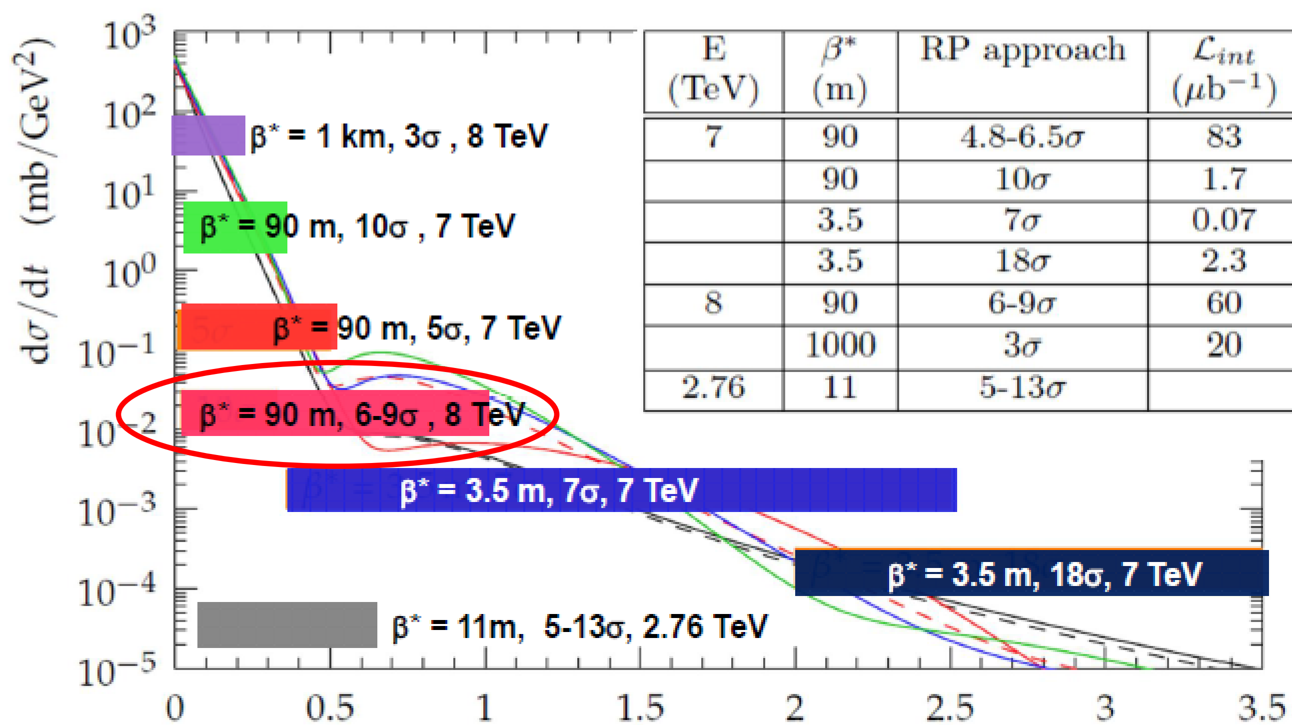
TOTEM – Experimental Setup at IP5





T1, T2: CSC and GEM Inelastic telescopes; RP: Roman Pots
[Details: JINST 3 (2008) S08007]. In this talk: TOTEM Roman Pots 220 m

TOTEM data taking



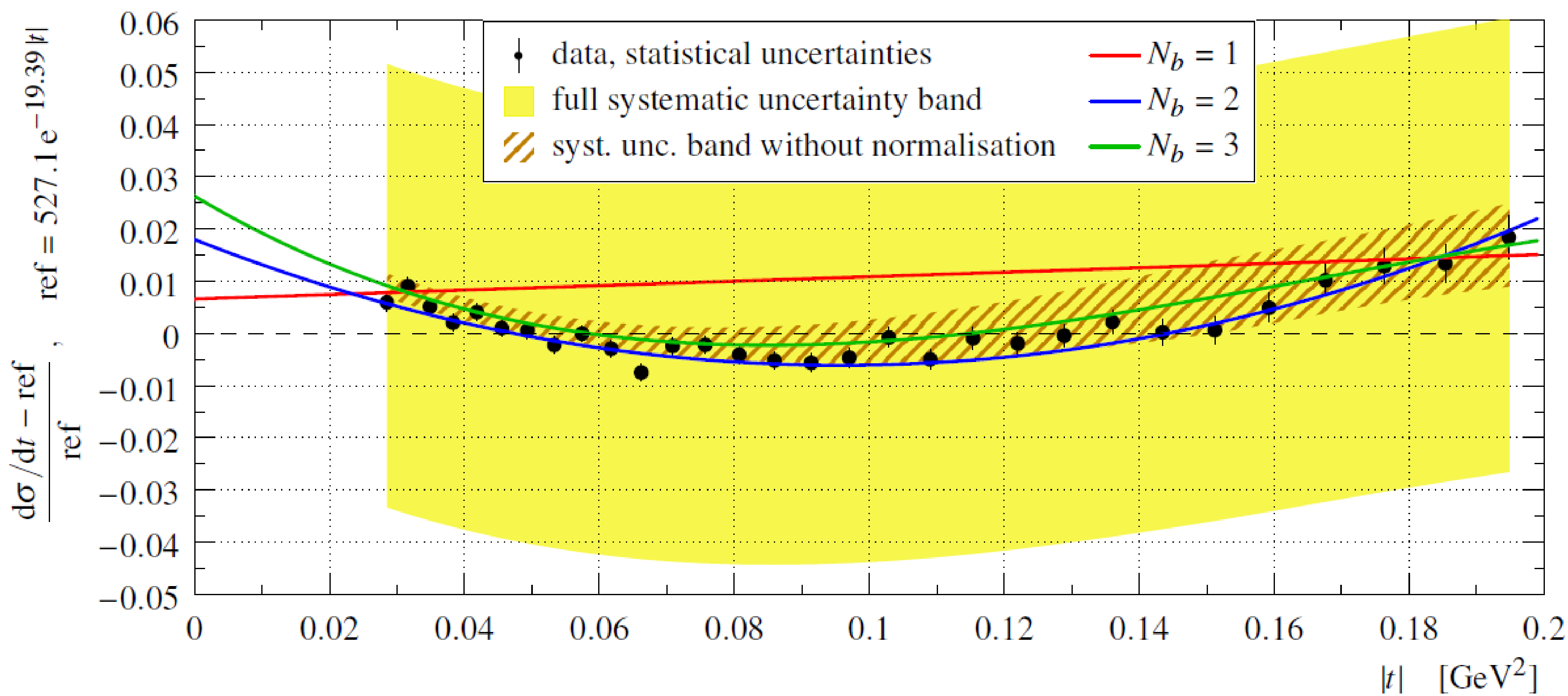
E (TeV)	β^* (m)	RP approach	$\mathcal{L}_{int}$ (μb^{-1})	t range (GeV^2)	Elastic events
7	90	4.8-6.5σ	83	$7 \cdot 10^{-3}$ - 0.5	1M
	90	10σ	1.7	0.02 - 0.4	14k
	3.5	7σ	0.07	0.36 - 3	66k
	3.5	18σ	2.3	2 - 3.5	10k
8	90	6-9σ	60	0.01 - 1	8M
	1000	3σ	20	$6 \cdot 10^{-4}$ - 0.2	0.4M
2.76	11	5-13σ		0.05-0.6	45k

Show results from data sets indicated by arrows

July 2012 data, **special** LHC run , $\beta^* = 90$ m, $\sqrt{s} = 8$ TeV

2 → 3 colliding bunch pair, 8 x 10^{10} p/bunch
Instantaneous $L \sim 10^{28}$ cm^{-2}s^{-1}
11 h data taking, RP-s at 9.5 σ_{beam}
Integrated $L \sim 735$ μb^{-1}
7.2 10^6 elastic events

Differential cross-section @ 8 TeV



$$\frac{d\sigma}{dt}(t) = \frac{d\sigma}{dt}\Big|_{t=0} \exp\left(\sum_{i=1}^{N_b} b_i t^i\right),$$

$$\chi^2 = \Delta^T V^{-1} \Delta,$$

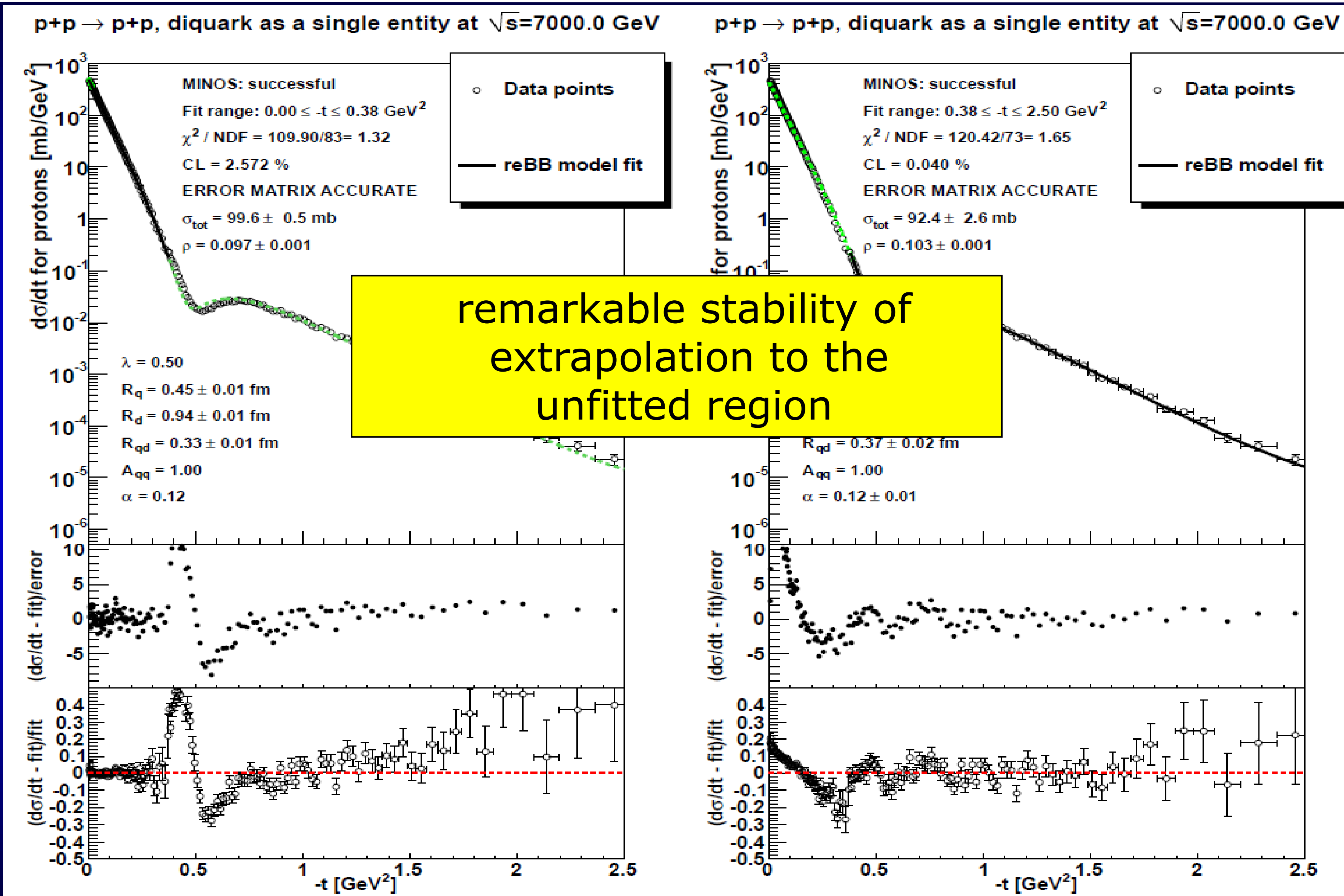
$$V = V_{stat} + V_{syst}$$

$$\Delta_i = \frac{d\sigma}{dt}\Big|_{\text{bin } i} - \frac{1}{\Delta t_i}\int_{\text{bin } i} f(t)\, dt,$$

N_b = 1 fits excluded. Relative to best exponential, a significant 7.2σ deviation found.

Backup slides – ReBB

ReBB model, fit range studies

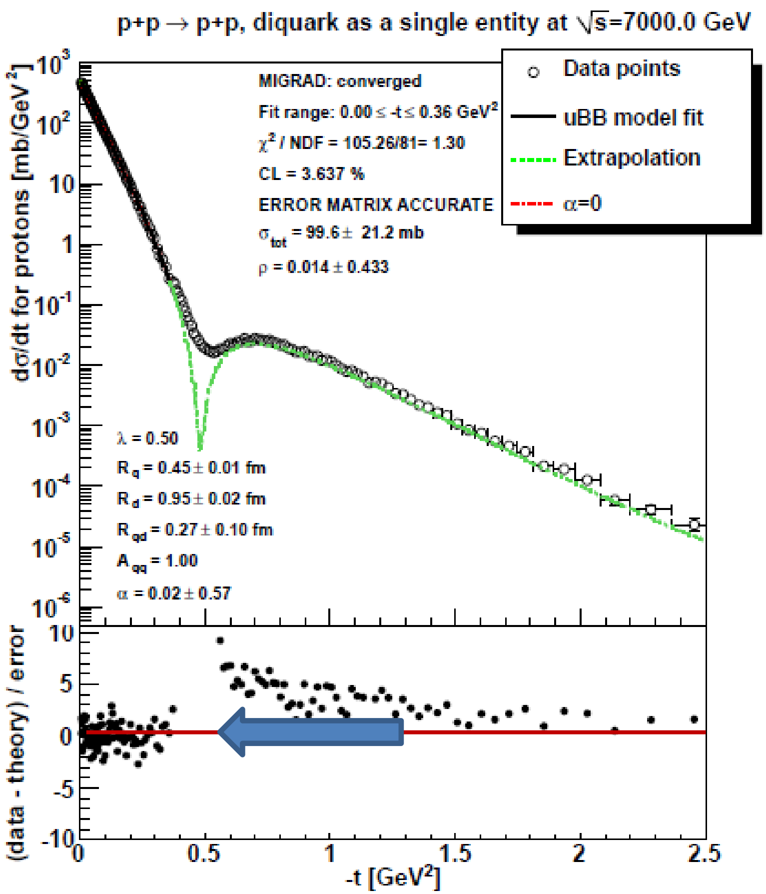


remarkable stability of extrapolation to the unfitted region

fit: 0.36≤ −t ≤ 2.5 GeV^2, OK

fit: 0 ≤ −t ≤ 2.5 GeV^2, ~ OK

Focusing reBB on the low-t region



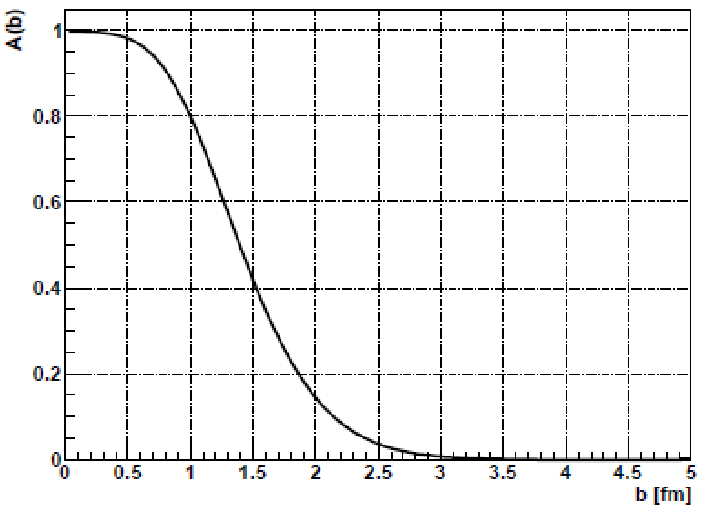


Figure 5: 0 - 0.36 GeV^2. ρ has large error, since the dip is not part of the fit.

Saturation is apparent if fit range is limited to $|t| < 0.36$ GeV^2

Focusing reBB on even lower –t region

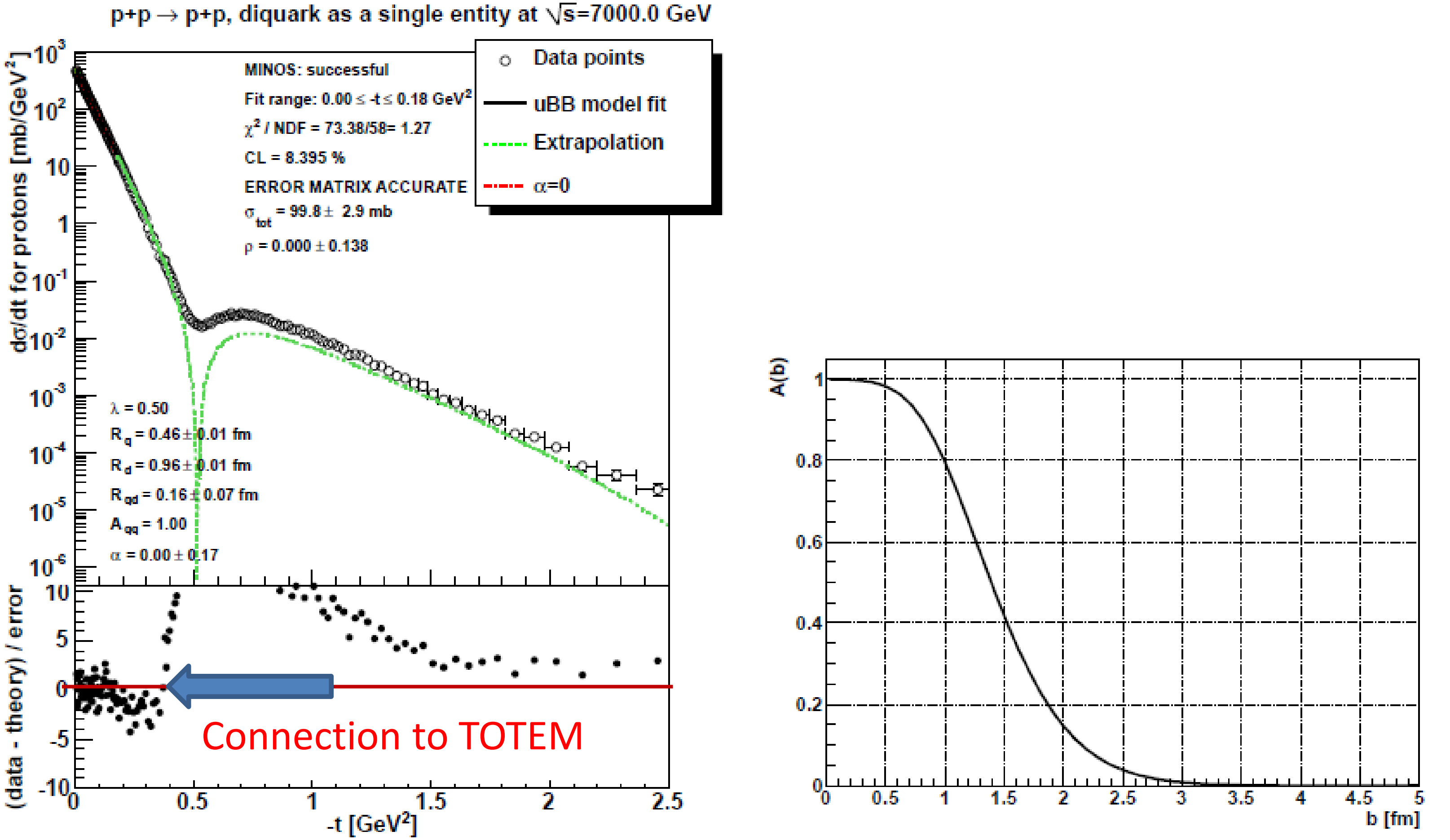


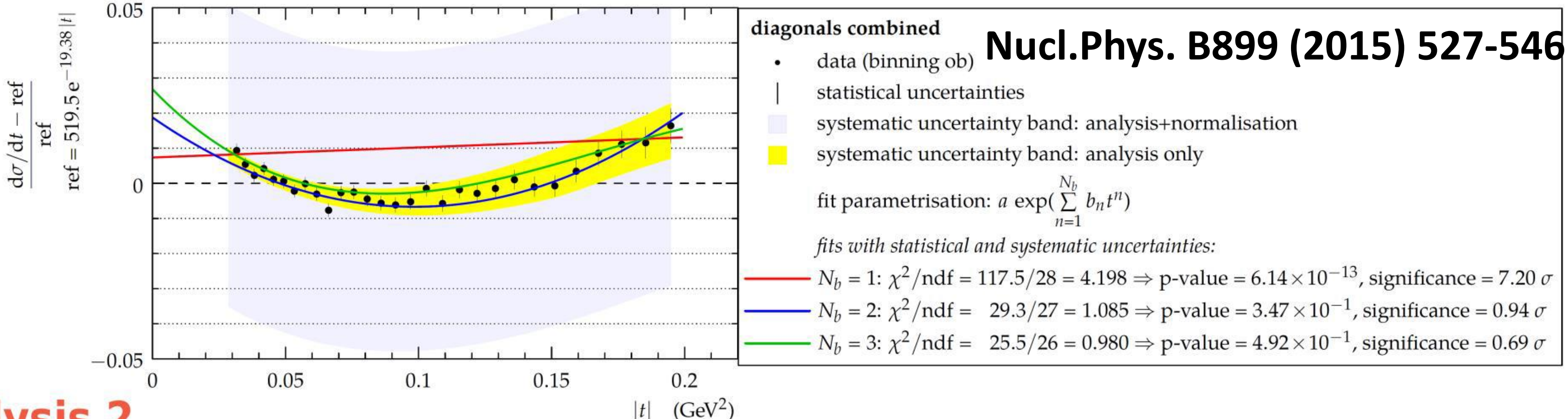
Figure 6: 0 - 0.18 GeV^2. ρ has large error, since the dip is not part of the fit.

Saturation still apparent, fit range $|t| < 0.18$ GeV^2

TOTEM 8 TeV pp data

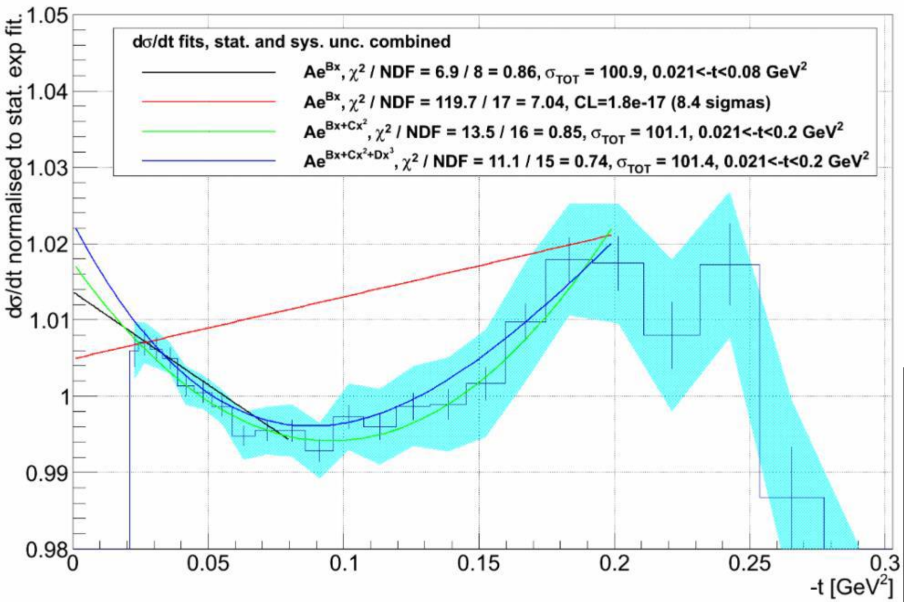
Analysis 1: fits $A \exp(b_1 t + b_2 t^2 + \ldots)$, N_b parameters in exponent

DS4



Nucl.Phys. B899 (2015) 527-546

Analysis 2



TOTEM pp data at 8 TeV
exponential shape
excluded at 7+ σ

new determination
σ_{tot} = (101.4 ± 2.0) mb

Theoretical support, from ISR
to LHC energies, unitarity
L. Jenkovszky and A. Lengyel,
arXiv:1410.4106

Non-exponential behaviour in ReBB

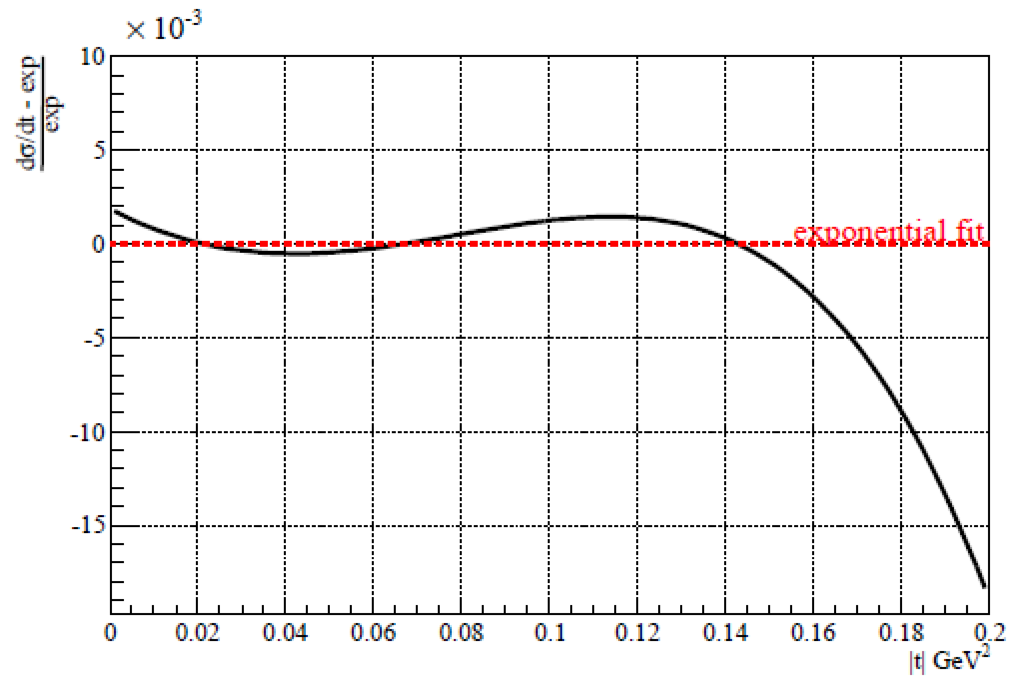
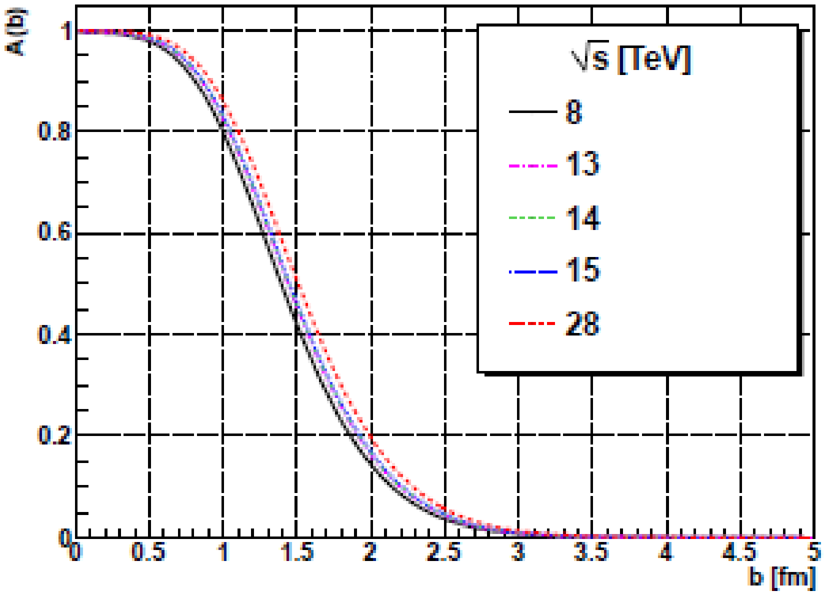
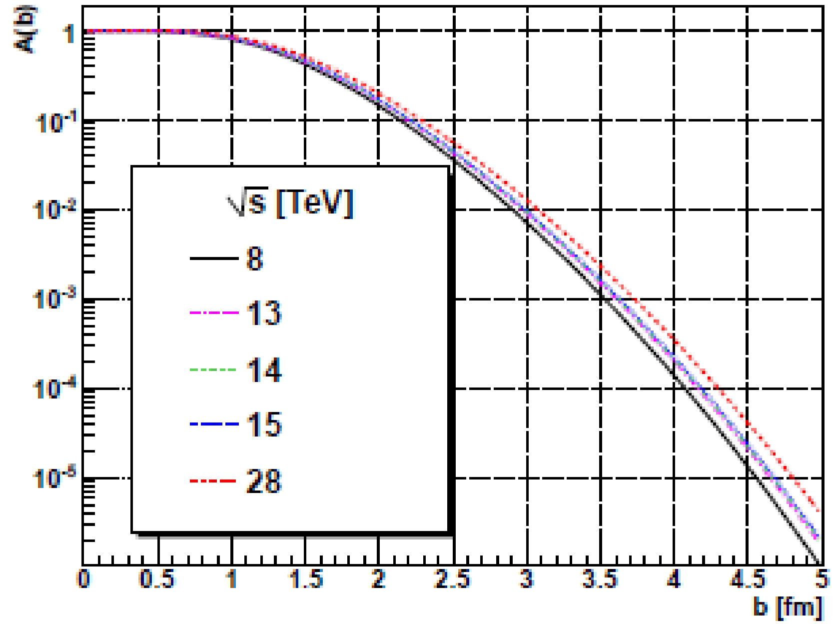


Fig. 5. The ReBB model, fitted in the $0.0 \leq |t| \leq 0.36$ GeV^2 range, with respect to the exponential fit of Eq. (33). In the plot only the $0.0 \leq |t| \leq 0.2$ GeV^2 range is shown. The curve indicates a significant deviation from the simple exponential at low $|t|$ values.

Similar non-exponential feature seen at 7 TeV as in 8 TeV TOTEM data

Predictions for the shadow profile

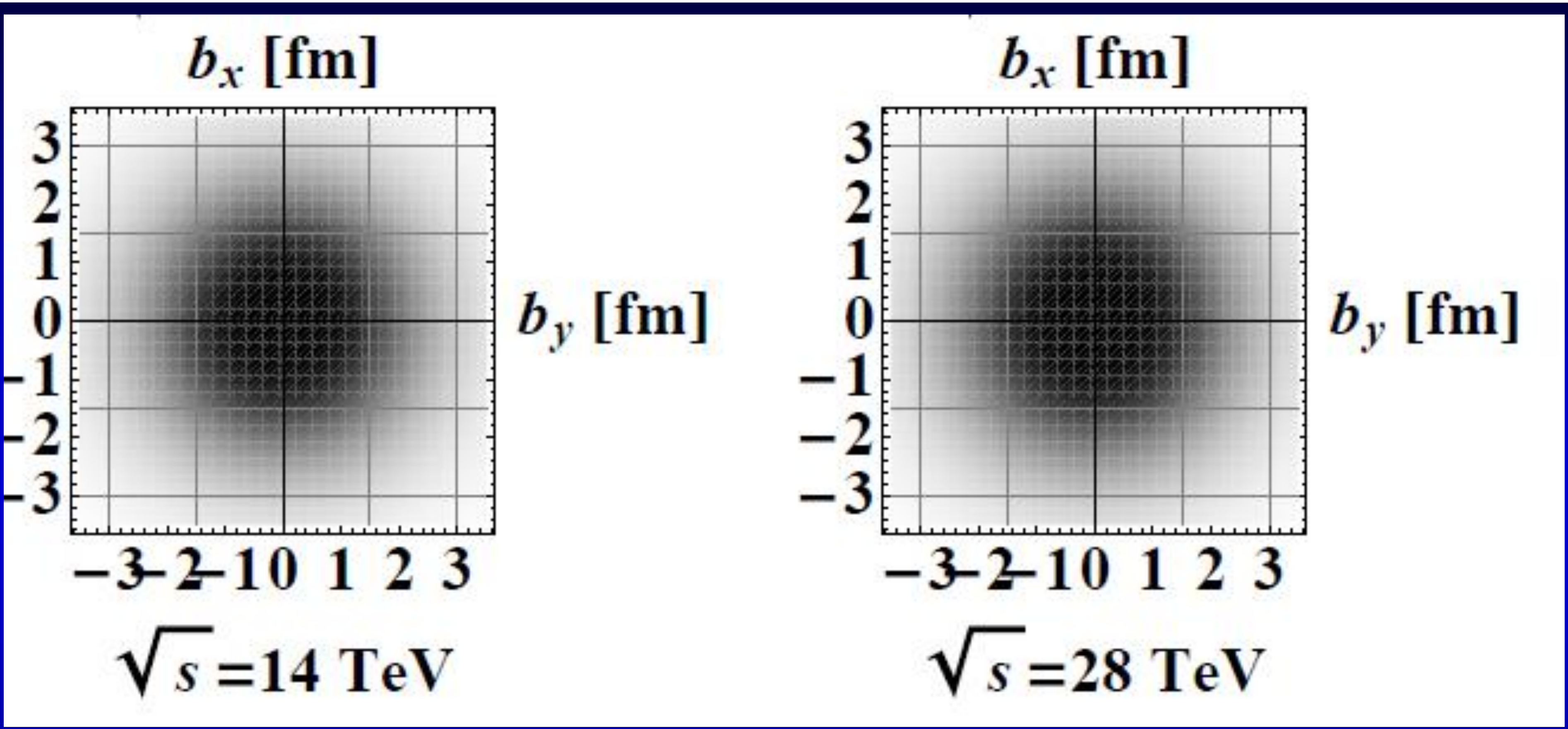




Blacker and Larger,
but not Edgier:
BnEL effect
at LHC energies

Similar to:
K.A. Kohara, T. Kodama,
E. Ferreira,
arXiv:1411.3518
but they also claim
an asymptotic BEL effect

Predictions for the shadow profile

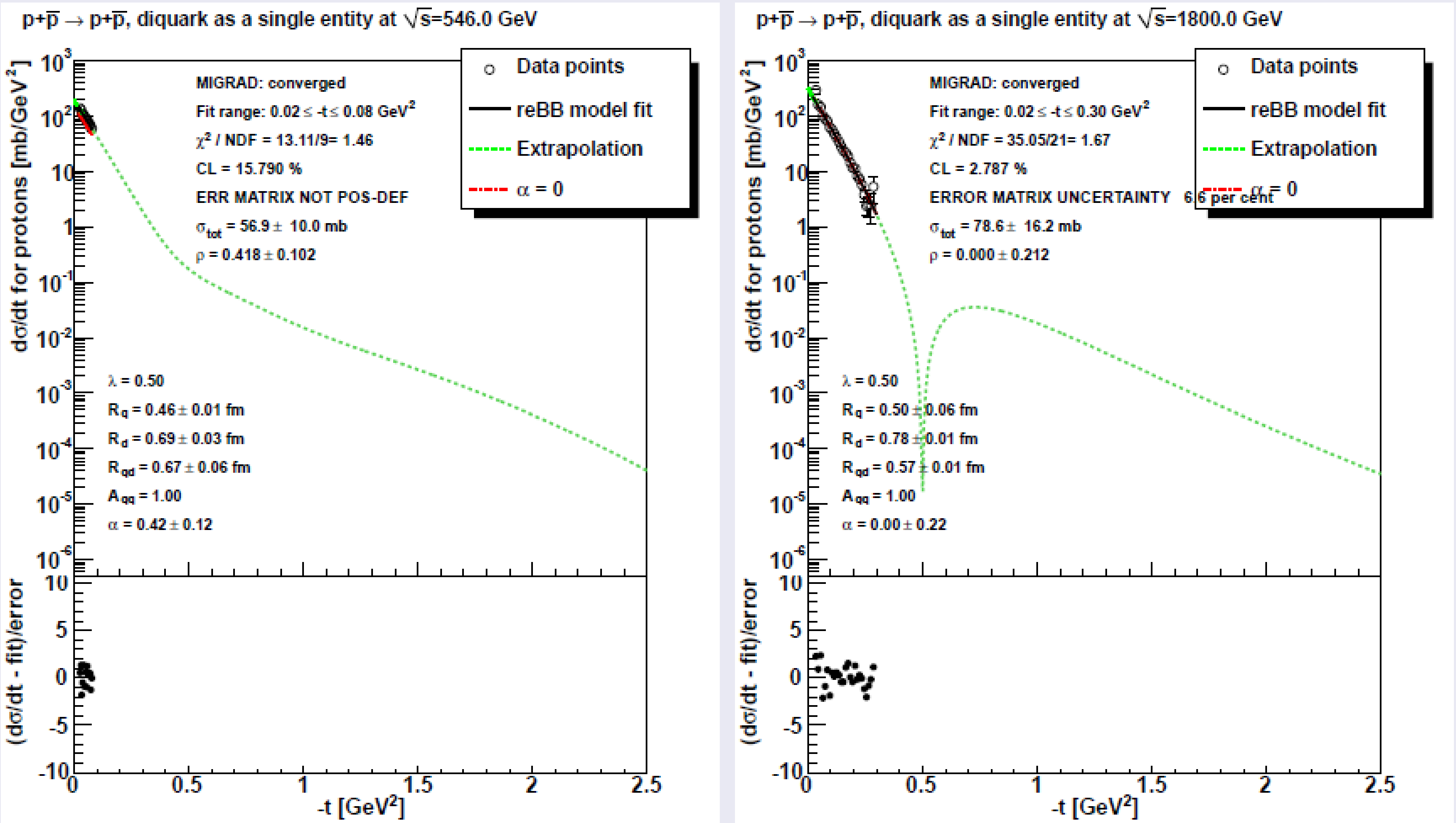


Blacker and Larger,
but not Edgier:
BnEL effect
at LHC energies

Results presented so far:
arxiv:1505.01415

New results: p$\bar{p}$ data with ReBB model

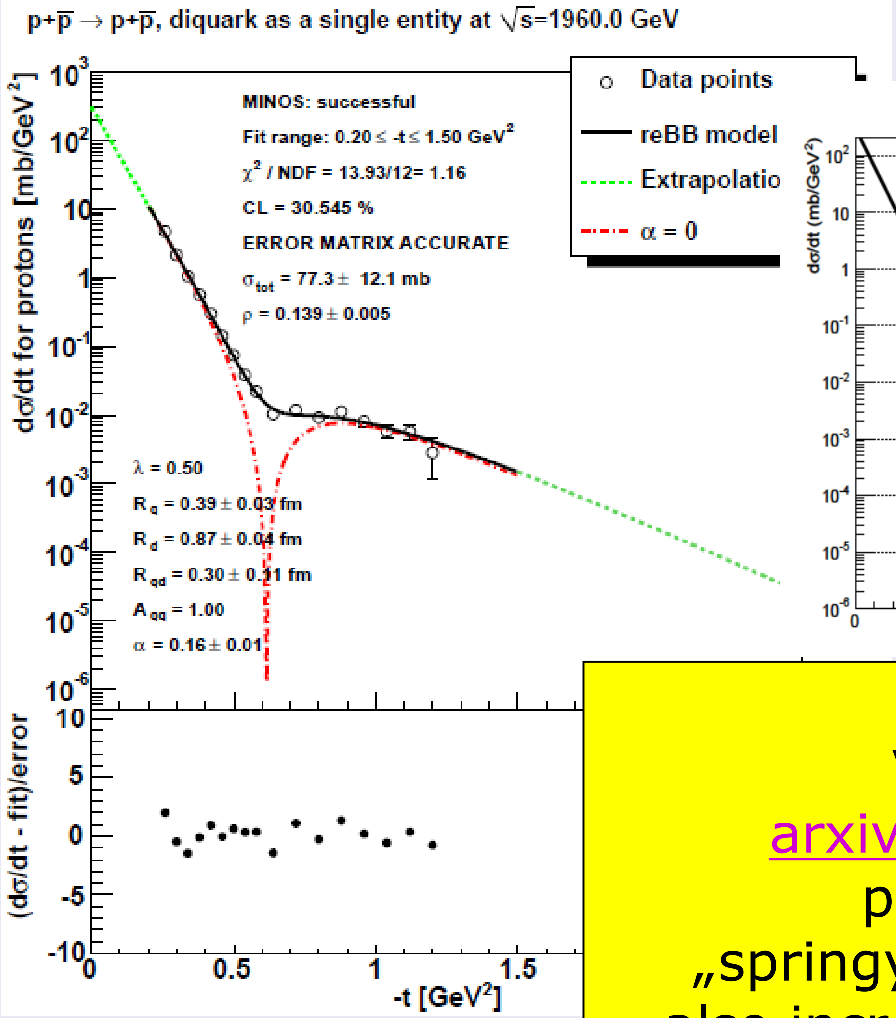
All the usual BB fit parameters are free ($\sqrt{s} = 546$ GeV, 1.8 TeV)

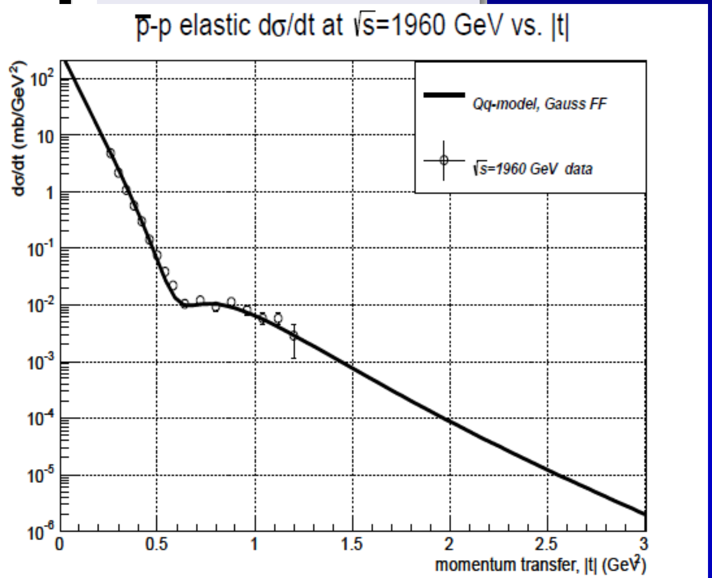


There are not enough points to pin down the shape.

Tevatron $p\bar{p}$ data with ReBB model

All the usual BB fit parameters are free (1.96 TeV)



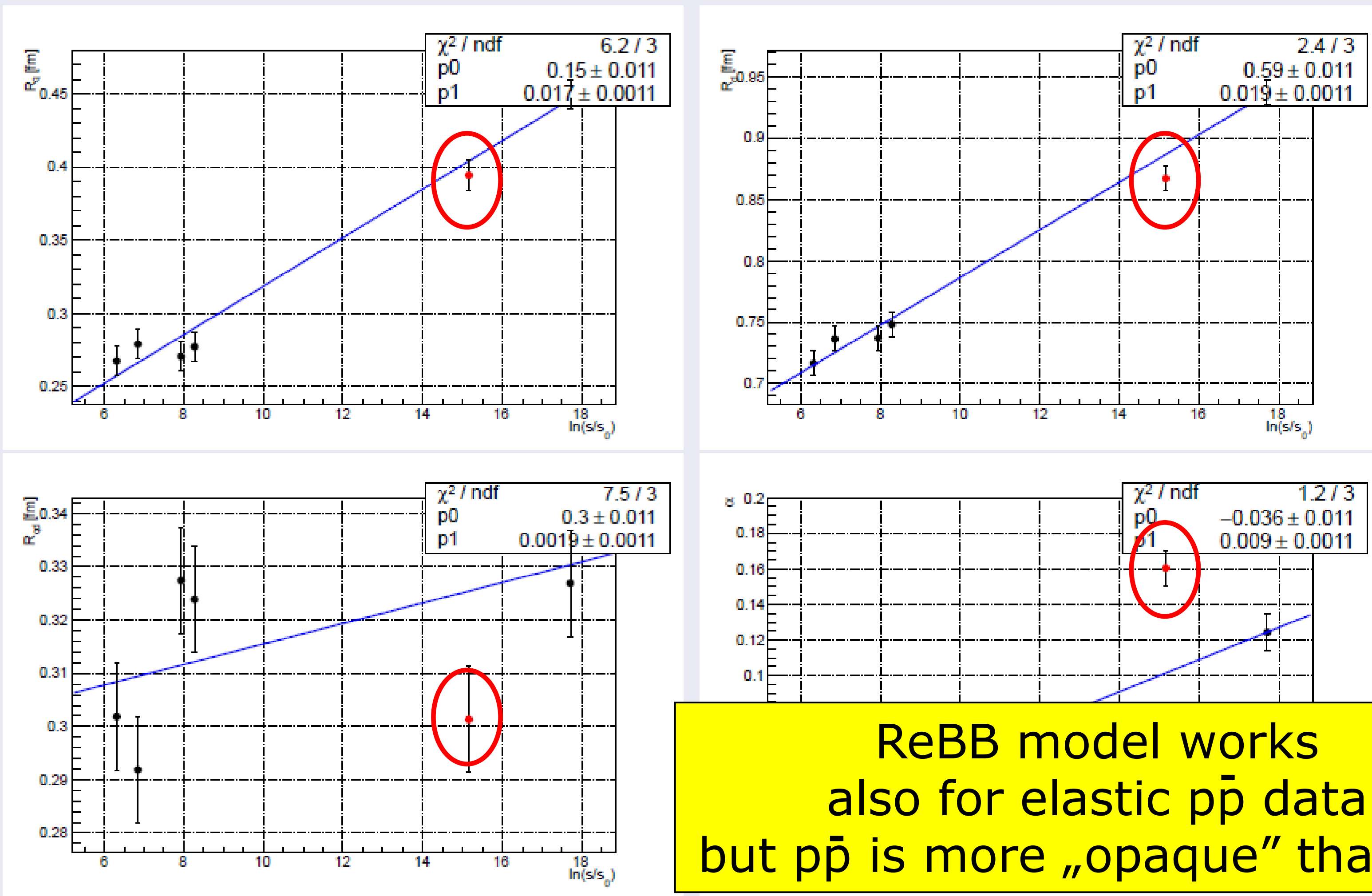


Similar to:,
V. Grichine,
arxiv.org:1404.5768
p = (q, d) in a
„springy" Pomeron picture
also increases real part of t_{el}

Ok.

Tevatron $p\bar{p}$ data trends ReBB model

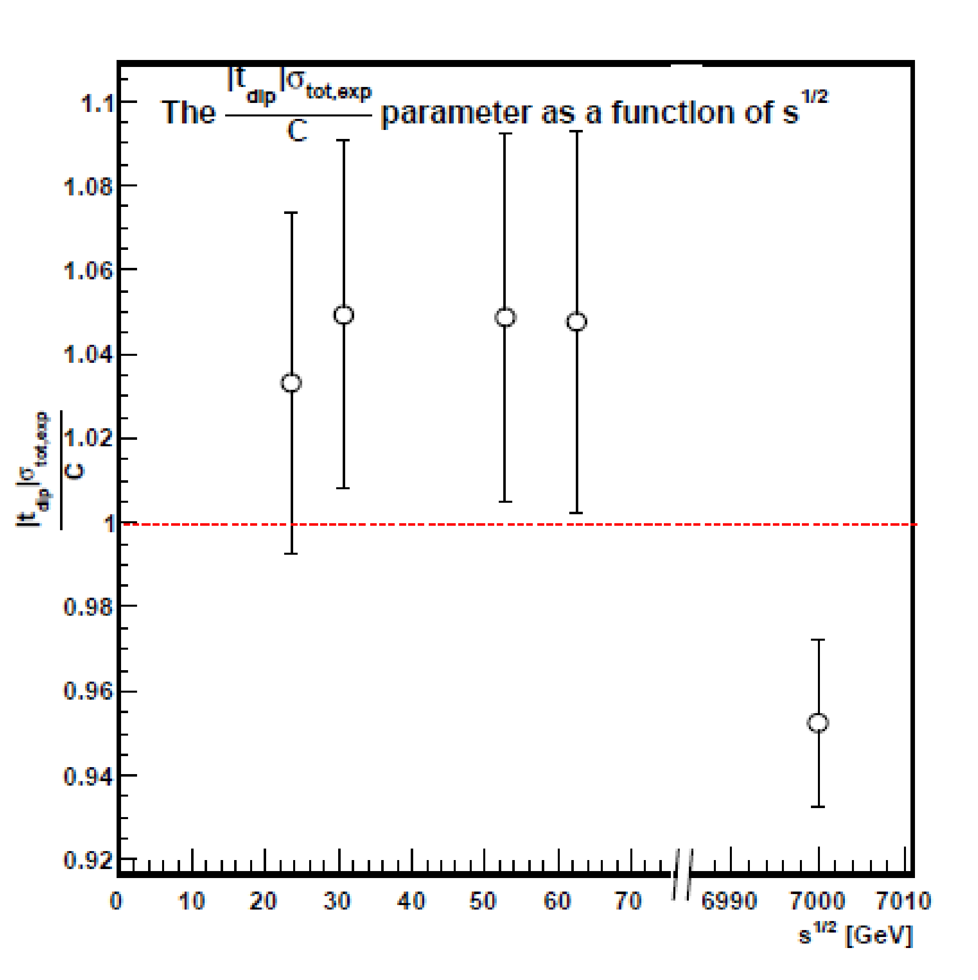
The good fit at $\sqrt{s} = 1.96$ TeV compared with the extrapolations based only on *pp* fits of our ReBB paper



ReBB model works
also for elastic $p\bar{p}$ data
but $p\bar{p}$ is more „opaque” than pp.

Backup slides – Discussion

Black disc limit?



Geometric scaling,
but not the black disc limit:

T. Cs. and F. Nemes
arXiv:1306.4217
Int. J. Mod. Phys. A (2014)

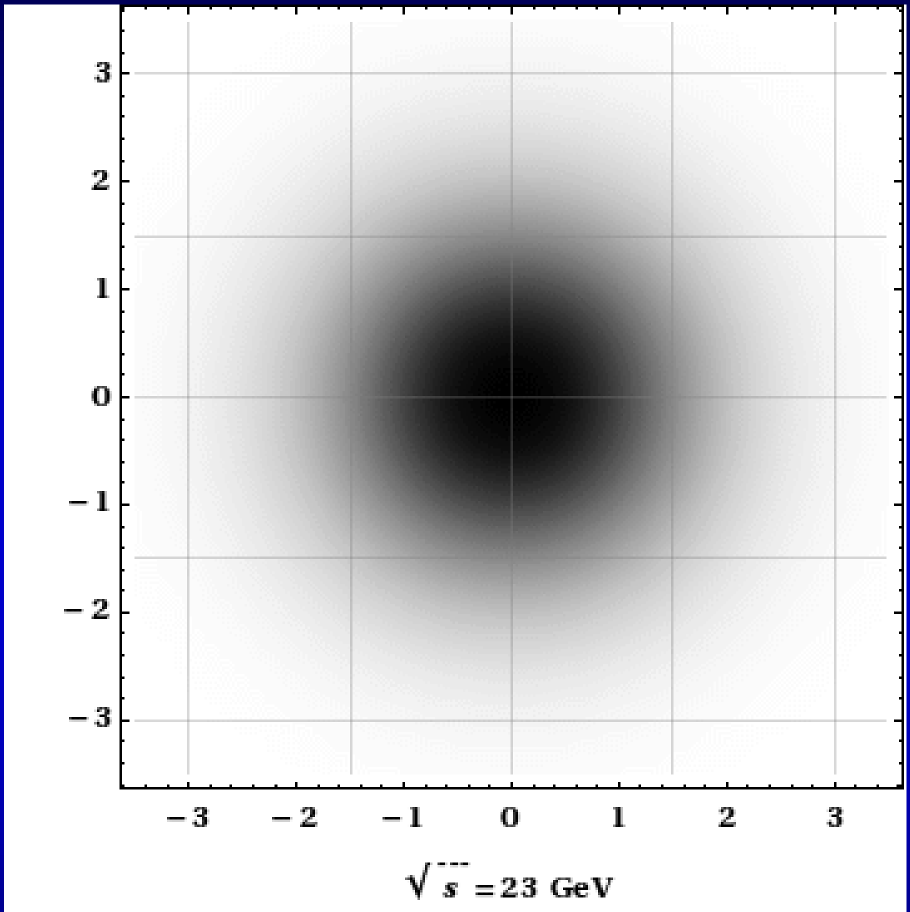
C(data) ~ 50 mb GeV^2
≠
C(black) ~ 36 mb GeV^2

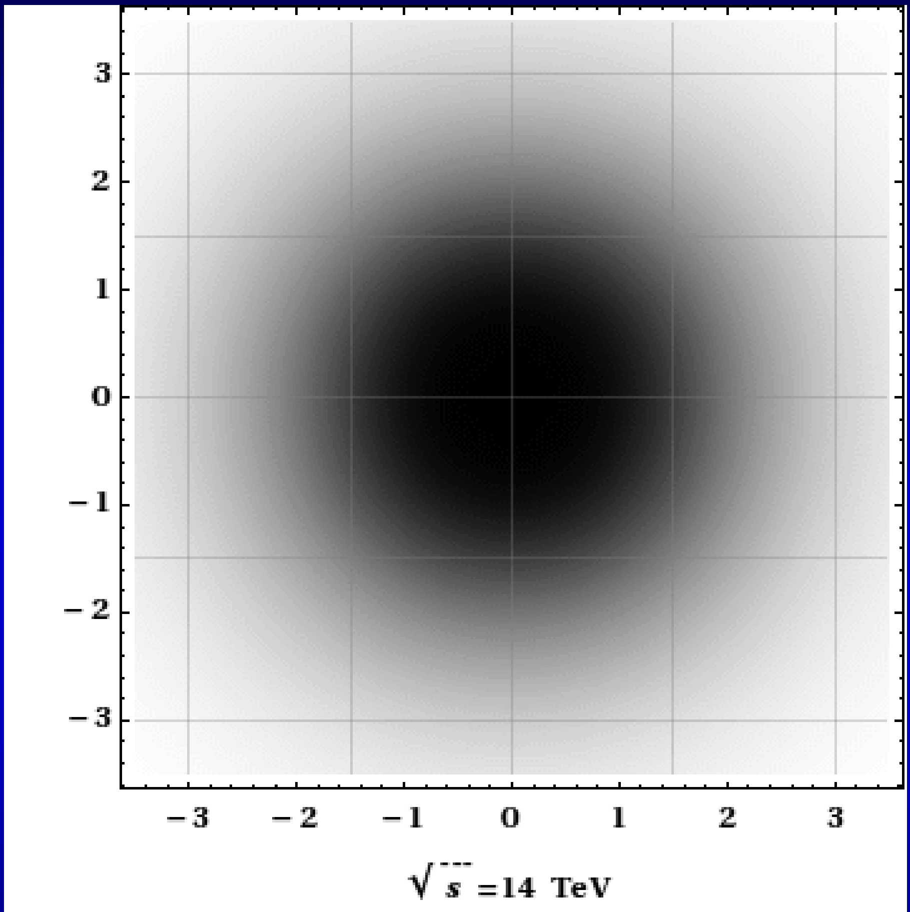
$$\frac{d\sigma_{black}}{dt} = \pi R^4 \left[\frac{J_1(qR)}{qR}\right]^2$$

$$\sigma_{tot,black} = 2\pi R^2 \,.$$

$$C_{black} = |t_{dip,black}| \cdot \sigma_{tot,black} = 2\pi j_{1,1}^2 (\hbar c)^2 \approx \; 35.9\,\mathrm{mb\ GeV}^2$$

Motivation: Is the proton a black disc?



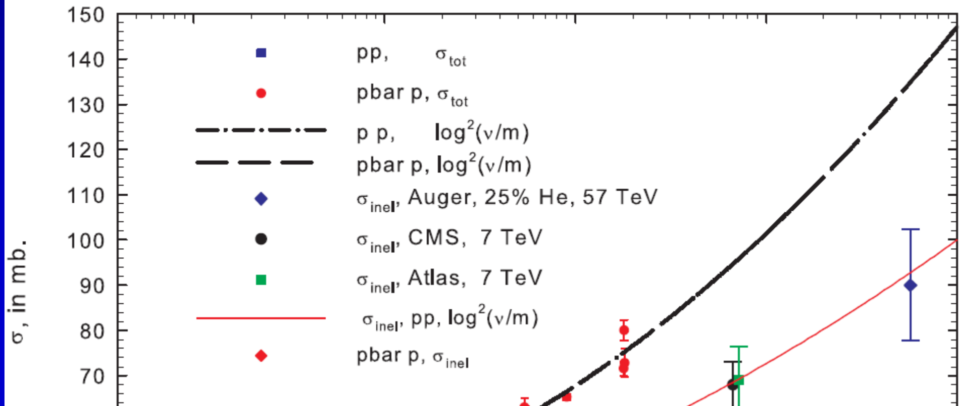


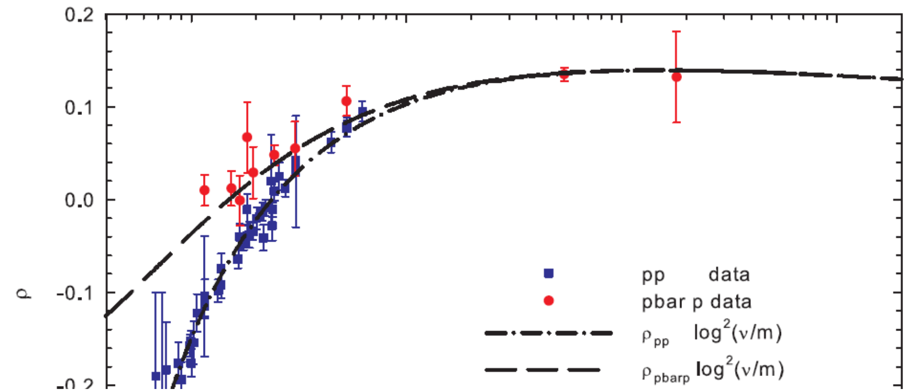
Recent papers by M. Block and F. Halsen address this topic :
Experimental confirmation: the proton is asymptotically a black disc,
arXiv:1109.2041, Phys. Rev. Lett. 107 (2011) 212002

Properties of a black disc

Properties of a black disk: In impact parameter space b, the elastic and total cross sections are given by

$$\sigma_{\rm el} = 4\int d^2b\, |a(b,s)|^2, \qquad \sigma_{\rm tot} = 4\int d^2b\, {\rm Im}\, a(b,s). \tag{7}$$



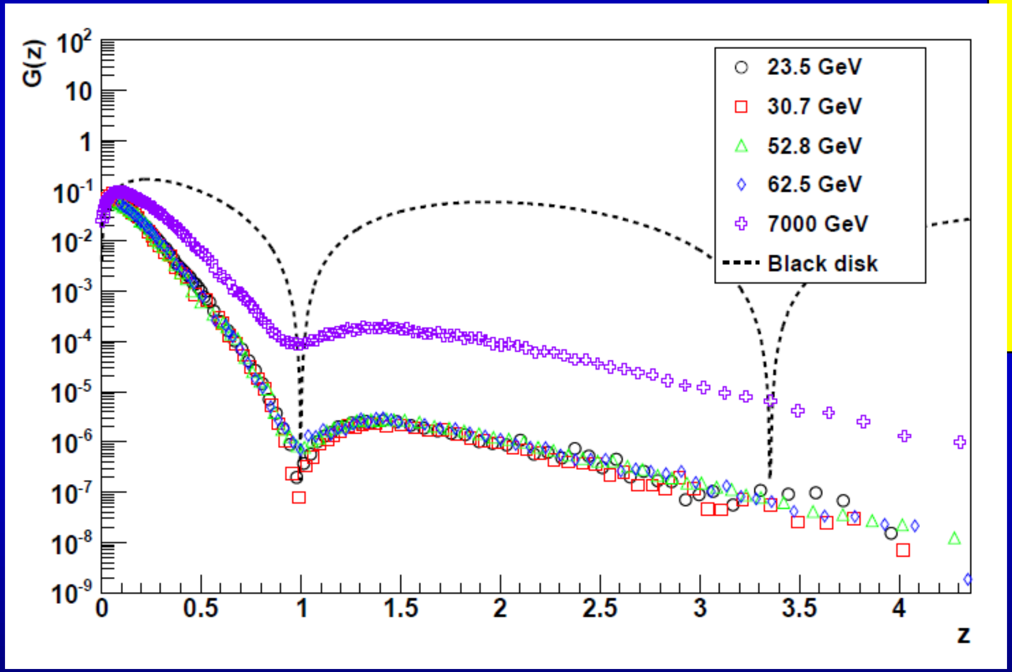


Conclusions: We find that the $\ln^2 s$ Froissart bound for the proton for $\sigma_{\rm tot}$ [7] and $\sigma_{\rm inel}$ [9] is saturated and that at infinite s, (1) the experimental ratio $\sigma_{\rm inel}/\sigma_{\rm tot} = 0.509 \pm 0.011$, compatible with the black disk ratio of 0.5 and (2) the forward scattering amplitude is purely imaginary. We thus conclude that the proton becomes an expanding black disk at sufficiently ultra-high energies that are probably never accessible to experiment. The theory for these bounds is predicated on the pillar stones of analyticity and unitarity, which have now been experimentally verified up to 57000 GeV. Further, since $\sigma_{\rm tot}$ has been extrapolated up from the Tevatron, we expect no new large cross section physics between 2000 and 57000 GeV.

Finally, the lowest-lying glueball mass is measured to be $M_{\rm glueball} = 2.97 \pm 0.03$ GeV. Reproducing these experimental results will be a task of lattice QCD.

arXiv:1109.2041, Phys. Rev. Lett. 107 (2011) 212002
arXiv:1208.4086, Phys.Rev. D86 (2012) 051504
arXiv:1409.3196

Black Disc (BD) limit?



Scaling but
not in the black disc limit:
T. Cs. and F. Nemes
arXiv:1306.4217
Int. J. Mod. Phys. A (2014)

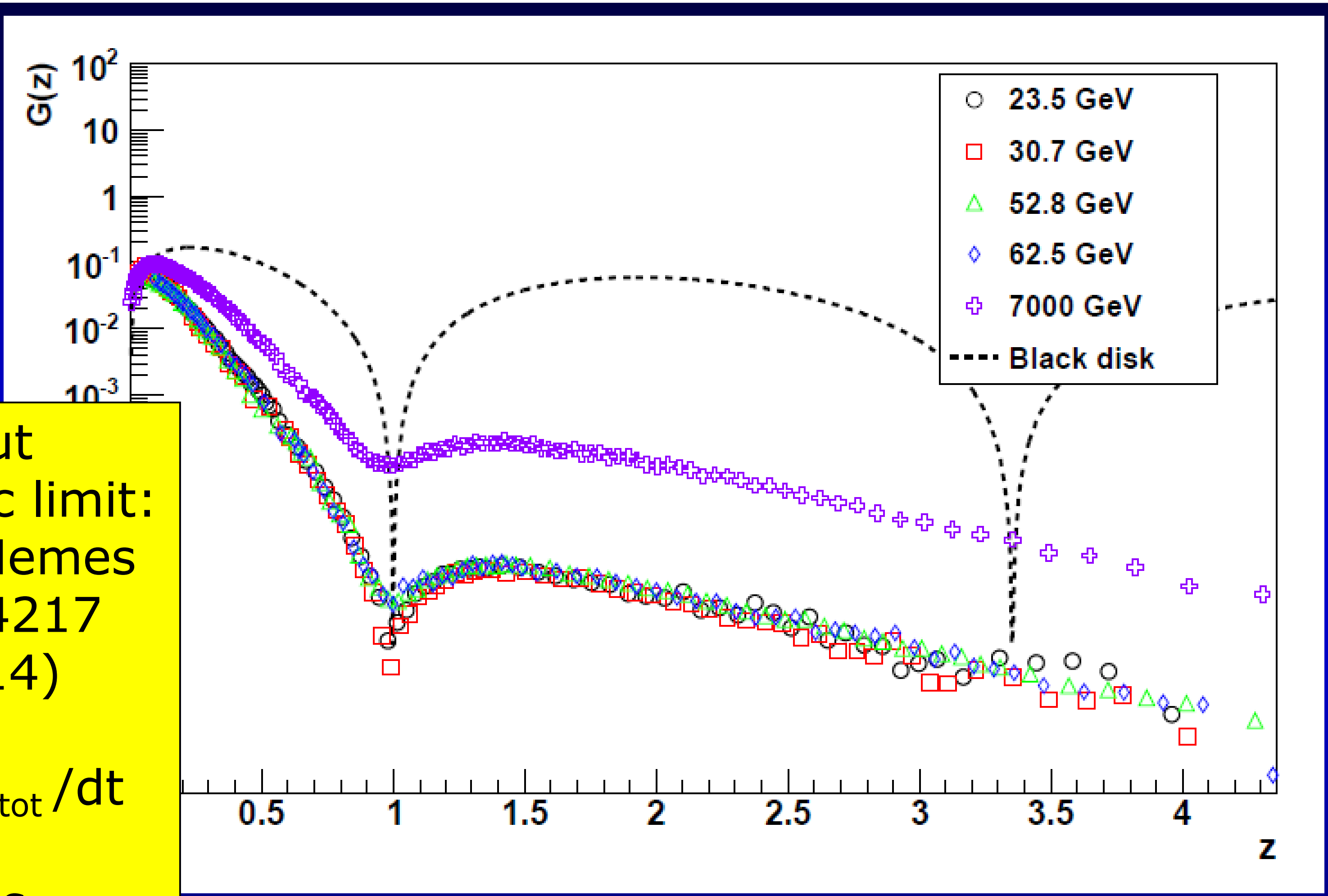
C(data) ~ 50 mb GeV^2
$\neq$
C(black) ~ 36 mb GeV^2

$$\frac{d\sigma_{black}}{dt} = \pi R^4 \left[\frac{J_1(qR)}{qR}\right]^2$$

$$\sigma_{tot,black} = 2\pi R^2 \,.$$

$$C_{black} = |t_{dip,black}| \cdot \sigma_{tot,black} = 2\pi j_{1,1}^2 (\hbar c)^2 \approx \; 35.9\,\text{mb GeV}^2$$

Geometric scaling, but not BD limit?



Scaling but
not a black disc limit:
T. Cs. and F. Nemes
arXiv:1306.4217
IJMPA (2014)

$G(z) = t\, d\sigma/\sigma_{tot}/dt$

plotted vs

$z = t/t_{dip}$