The Nobel Prize in Physics 2016

David J. Thouless
Prize share: 1/2

Born 1934 in Bearsden, UK. Ph.D. 1958 from Cornell University, Ithaca, NY, USA. Emeritus Professor at University of Washington, Seattle, WA, USA.

F. Duncan M. Haldane
Prize share: 1/4

Born 1951 in London, UK. Ph.D. 1978 from Cambridge University, UK. Eugene Higgins Professor of Physics at Princeton University, NJ, USA.

J. Michael Kosterlitz
Prize share: 1/4

Born 1942 in Aberdeen, UK. Ph.D. 1969 from Oxford University, UK. Harrison E. Farnsworth Professor of Physics at Brown University, Providence, RI, USA.

"for theoretical discoveries of topological phase transitions and topological phases of matter"

Strange phenomena in matter's flatlands

The Kosterlitz-Thouless phase transition

J. Phys. C: Solid State Phys., Vol. 5, 1972. Printed in Great Britain. © 1972

LETTER TO THE EDITOR

Long range order and metastability in two dimensional solids and superfluids

J M KOSTERLITZ and D J THOULESS

Department of Mathematical Physics, University of Birmingham, Birmingham B15 2TT

J. Phys. C: Solid State Phys., Vol. 6, 1973. Printed in Great Britain. © 1973

Ordering, metastability and phase transitions in two-dimensional systems

J M Kosterlitz and D J Thouless

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2D XY-model

$$H_{XY} = -J \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j) \qquad 0 \le \theta_i < 2\pi$$

i) Direction of an XY-spin

ii) Phase of a superfluid $\psi = \sqrt{\rho_s} e^{i\theta}$

Mermin-Wagner theorem: there is no spontaneous magnetization at T>0

Numerical works showing phase-transition at finite T

Theoretical considerations

Continuum limit: $H_{XY} = \frac{J}{2} \int d^2r \, (\vec{\nabla}\theta(\vec{r}))^2$ extend the range: $-\infty < \theta < \infty$

Gaussian integration: $\langle e^{i(\theta(\vec{r})-\theta(\vec{0}))} \rangle \sim \left(\frac{a}{r}\right)^{\frac{k_B T}{2\pi J}}$

Not correct in the high-temperature case

One can not ignore the periodicity of θ

Kosterlitz and Thouless solution:

Topological, vortex-like configurations, with vorticity: $v = \frac{1}{2\pi} \oint_C d\vec{r} \cdot \vec{\nabla}\theta(\vec{r})$

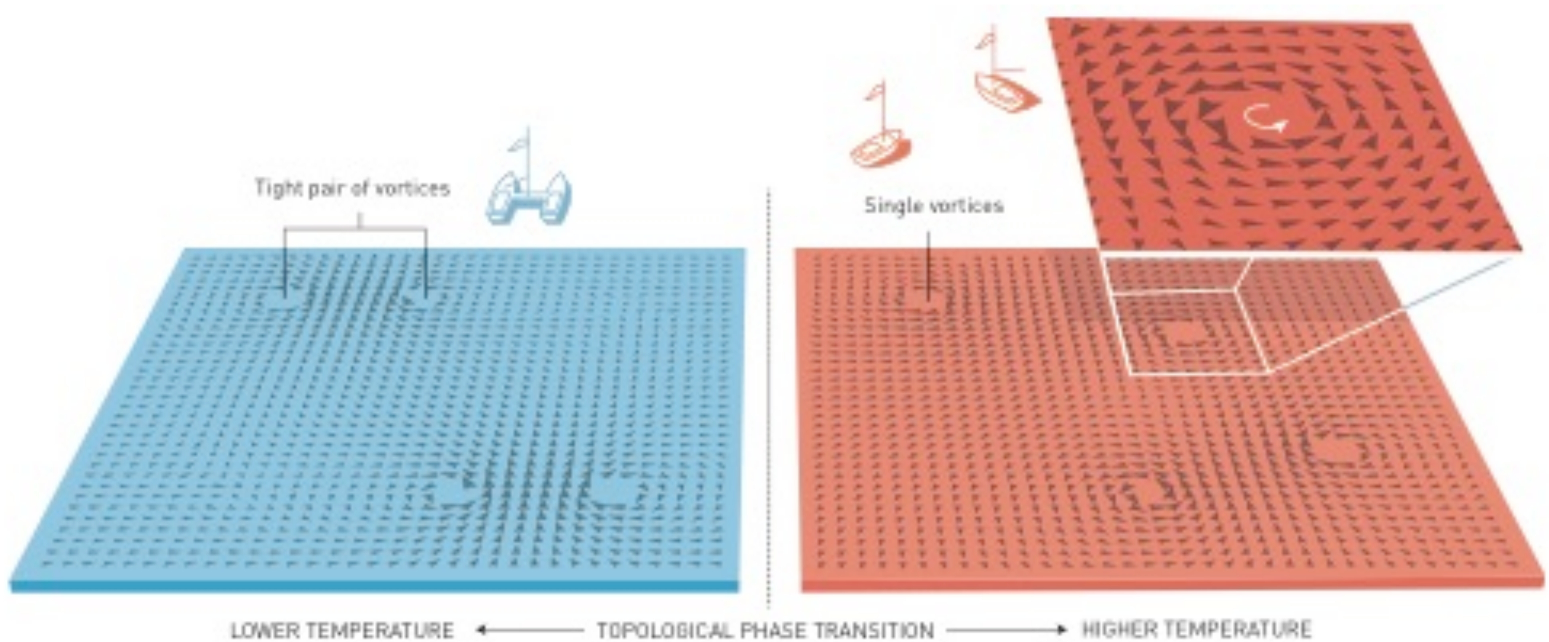
For $v = \pm 1$ and $|\vec{\nabla}\theta(\vec{r})| = 1/r$

Energy of a single vortex: $E_v = \frac{J}{2}\int d^2r \left(\frac{1}{r}\right)^2 = J\pi \ln\frac{L}{a}$

Energy of a vortex-antivortex pair: $E_{va} = 2J\pi \ln(r/a)$

Free energy for a single vortex: $F = E - TS = J\pi \ln\left(\frac{L}{a}\right) - Tk_B \ln\left(\frac{L^2}{a^2}\right)$

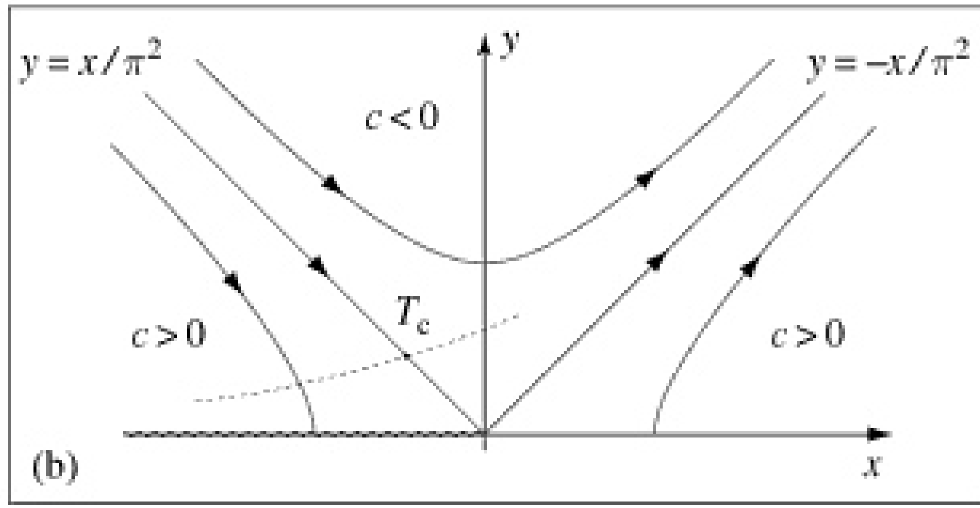
Kosterlitz-Thouless critical temperature: $T_{KT} = J\pi/2k_B$



Further developments:

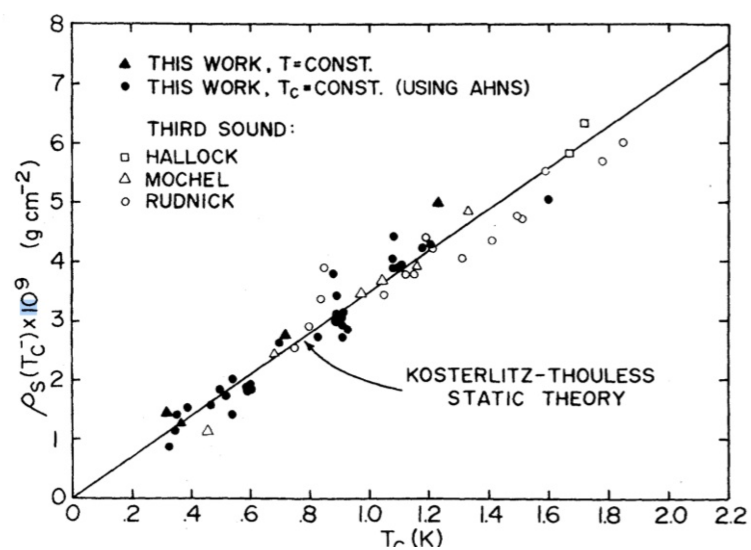
Kosterlitz RG equations:

$$\frac{dt}{d\ell} = 4\pi^3 y^2$$
$$\frac{dy}{d\ell} = 4ty/\pi$$
$$t = x = T - T_c$$
$$y = y_0 e^{-\pi J/k_B T}$$



Nelson and Kosterlitz: "universal jump" of the superfluid density at the KT-transition

$$\rho(T_c) = T_c \frac{2}{\pi} \frac{m^2 k_B}{\hbar^2}$$



Quantum spin chains and symmetry-protected topological phases of matter

Volume 93A, number 9 PHYSICS LETTERS 14 February 1983

CONTINUUM DYNAMICS OF THE 1-D HEISENBERG ANTIFERROMAGNET: IDENTIFICATION WITH THE O(3) NONLINEAR SIGMA MODEL

F.D.M. HALDANE
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VOLUME 50, NUMBER 15 PHYSICAL REVIEW LETTERS 11 APRIL 1983

Nonlinear Field Theory of Large-Spin Heisenberg Antiferromagnets: Semiclassically Quantized Solitons of the One-Dimensional Easy-Axis Néel State

F. D. M. Haldane
Department of Physics, University of Southern California, Los Angeles, California 90089

Antiferromagnetic Heisenberg chain

$$H = J \sum_{\langle ij \rangle} \mathbf{S_i S_j}$$

For S=1/2 Bethe Ansatz-solution: it is gapless

For S>1/2 it was a general believe, that the spectrum is gapless

Haldane: for large-S derived an effective Hamiltonian:

$$S_{NLS} = \frac{1}{2g} \int dt dx \left(\frac{1}{v} (\partial_t \vec{n})^2 - v(\partial_x \vec{n})^2 \right)$$

$\vec{n}$ is a unit vector, v is the spin wave velocity and $g = 2/S$

This is the O(3) non-linear sigma model

It is gapped, which should hold for the AF Heisenberg chain for any value of the spin

It is in contradiction with the exact result for S=1/2

Haldane solution: there are large fluctuations, which depend on the value of S

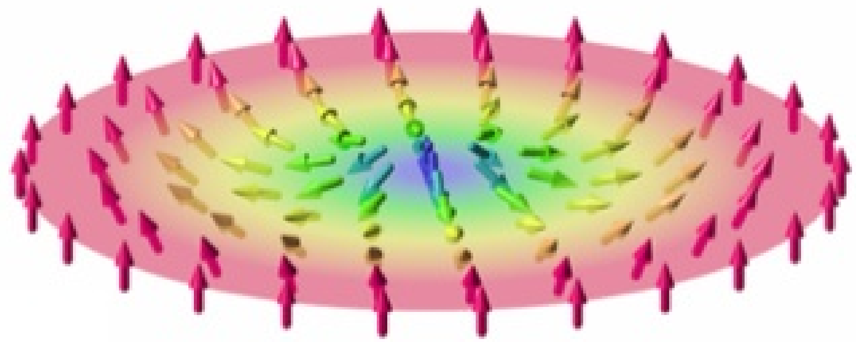
Additional topological θ-term

$$S_{top} = i\frac{\theta}{4\pi}\int d^2x\ \vec{n}\cdot(\partial_1\vec{n}\times\partial_2\vec{n}) \qquad \theta = 2\pi S,$$

winding number:

$$Q = \frac{1}{4\pi}\int d^2x\,\vec{n}\cdot(\partial_1\vec{n}\times\partial_2\vec{n})$$

is an integer



Q=1

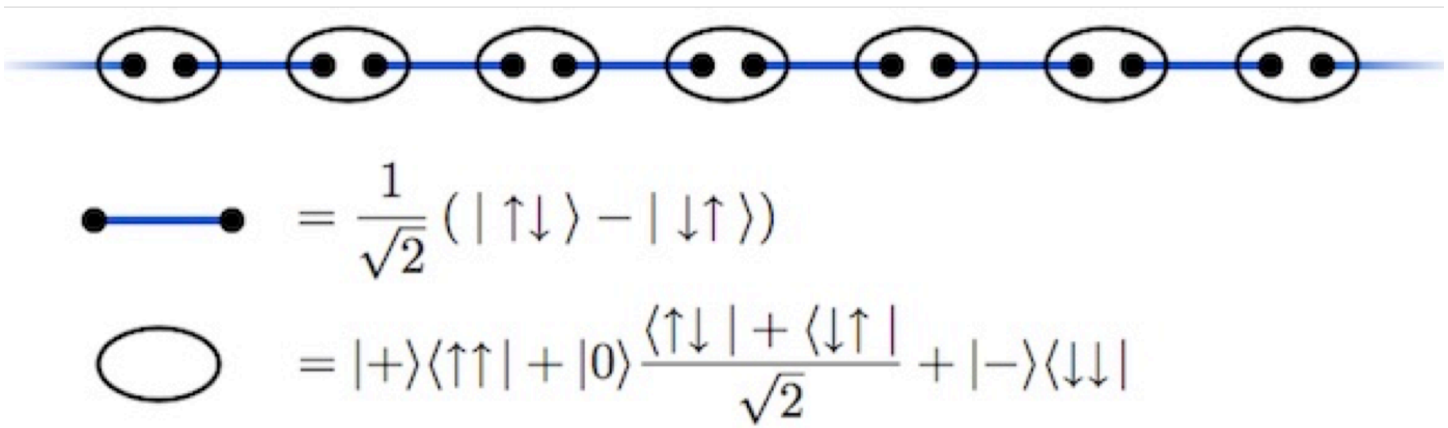
There are phase factors: $e^{i2\pi SQ}$

These are irrelevant for S=integer, but will cause a vanishing gap for S=half integer

AKLT (Affleck, Kennedy, Lieb, Tasaki) model

$$H_{AKLT} = \sum_{\langle ij \rangle} \mathbf{S_i S_j} + \frac{\mathbf{1}}{\mathbf{3}} \left(\mathbf{S_i S_j}\right)^{\mathbf{2}}$$

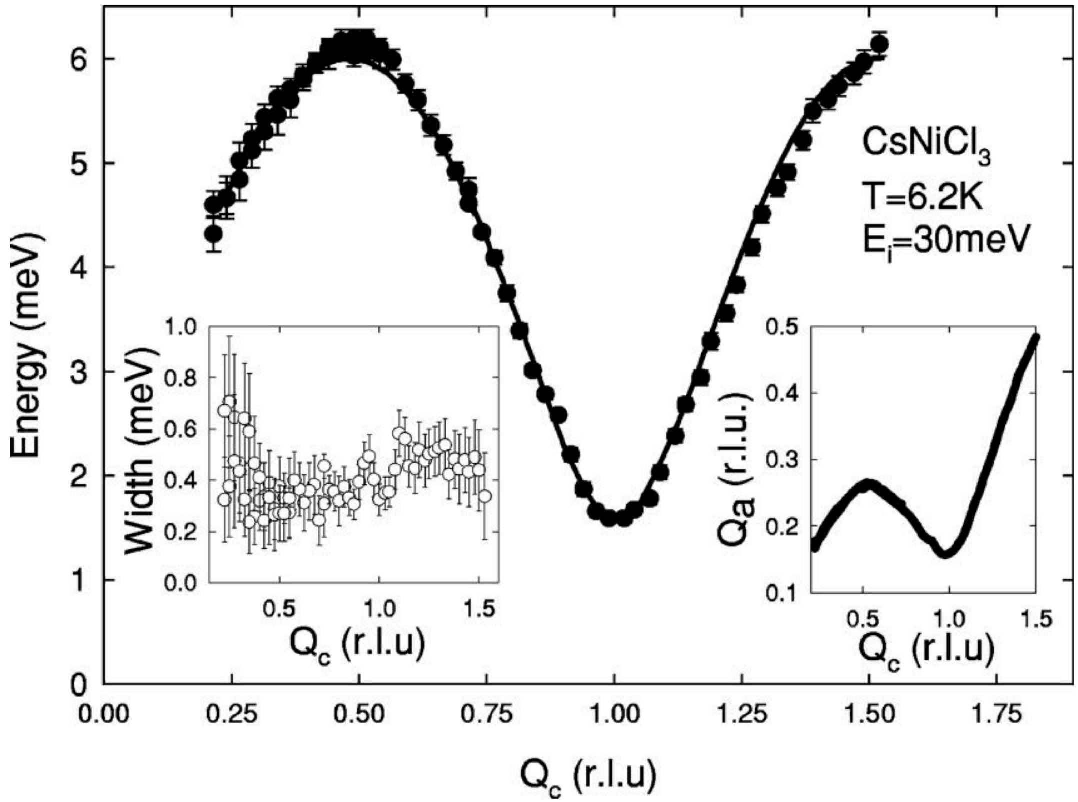
Valence bond solid representation in terms of S=1/2 spins:



It has a Haldane-gap

For free chains spin-1/2 degrees of freedom at the boundary

Experimental verification of the Haldane-gap in $CsNiCl_3$



Kenzelmann et al, Phys. Rev. B66, 024407 (2002)

The Haldane-phase is the prototype of

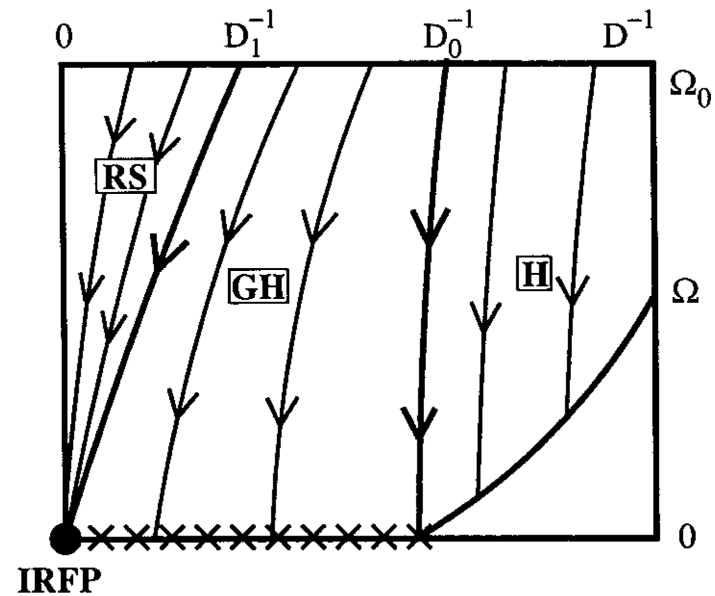
Symmetry protected topological states

String order parameter

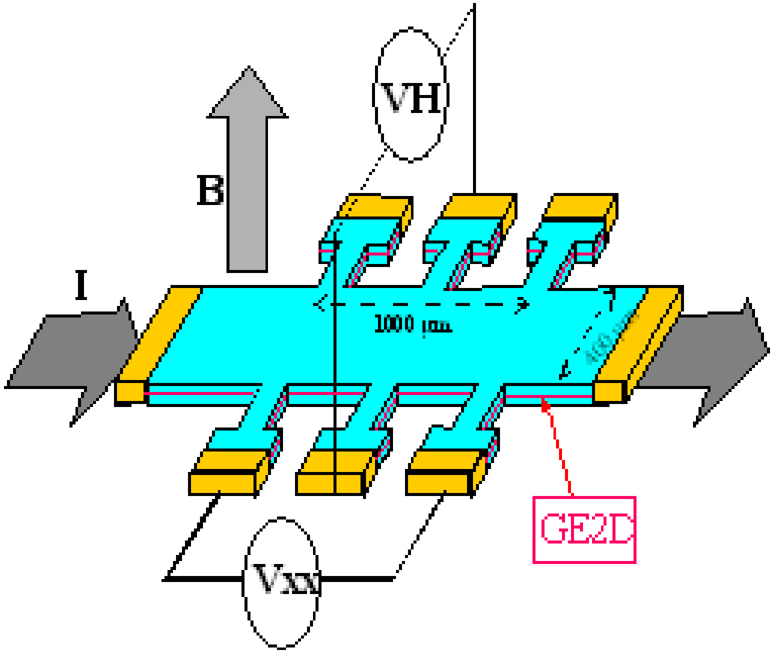
$$O^z(r) = -\langle S_l^z \exp[i\pi(S_{l+1}^z + S_{l+2}^z + \cdots + S_{l+r-1}^z)] S_{l+r}^z \rangle,$$

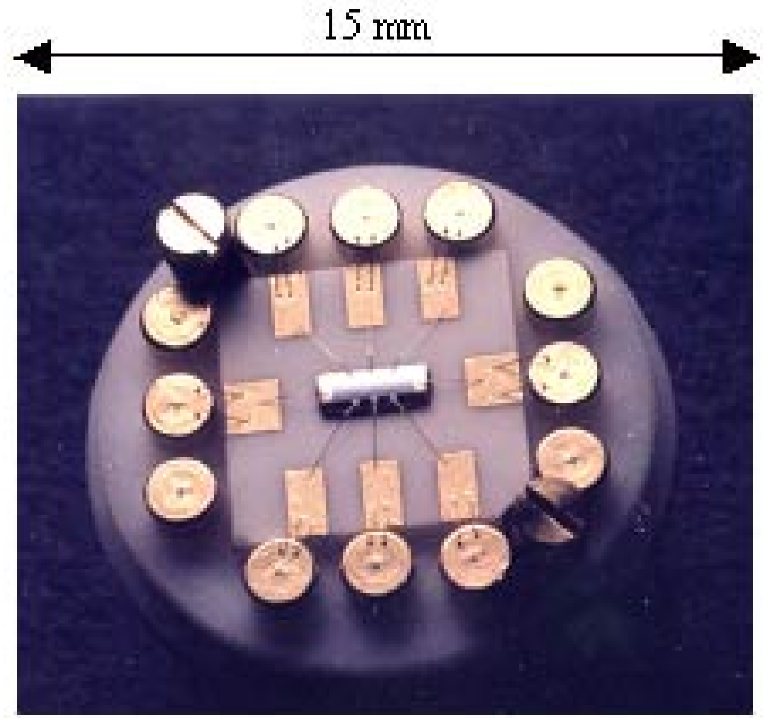
Remains intact in the presence of small perturbations

Example: bond disorder of strength D



Quantum Hall Effect (1980): resistance measurements on a 2-dimensional electron gas

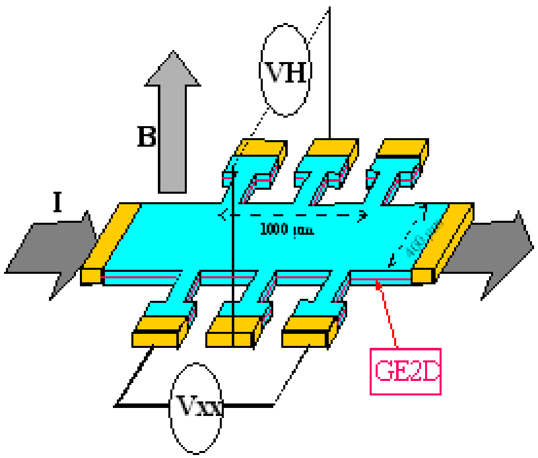


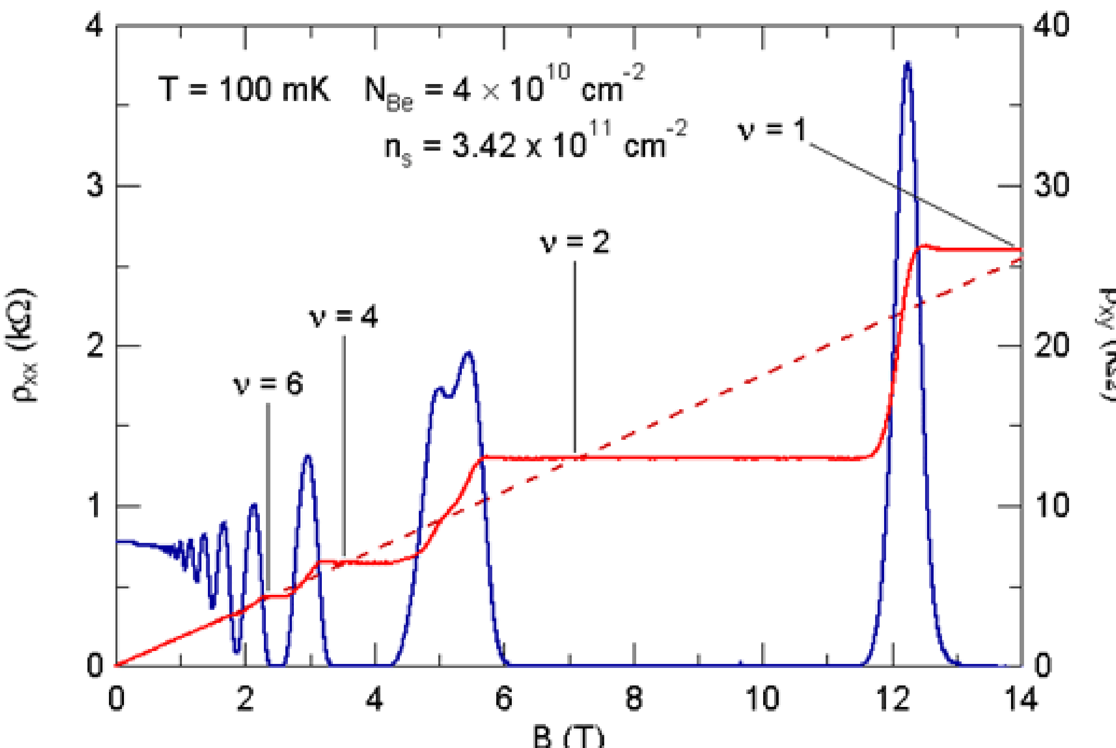


1980, von Klitzing (1985)

Source: Laboratoire national de metrologie et d'essais, French Government

Quantum Hall Effect (1980): Thermodynamic phase without order parameter

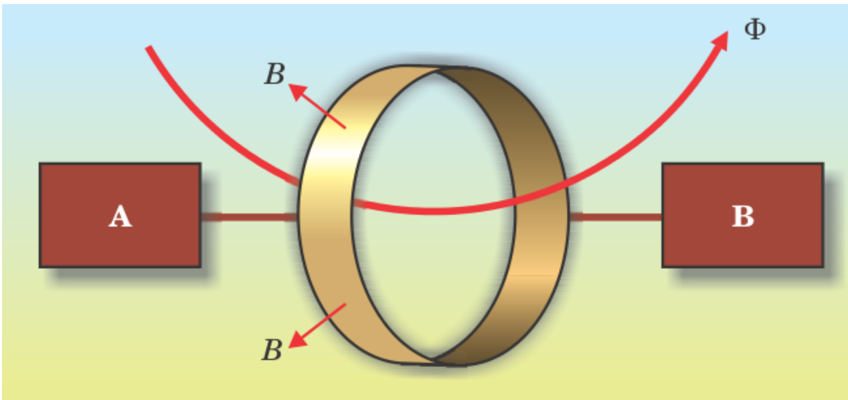


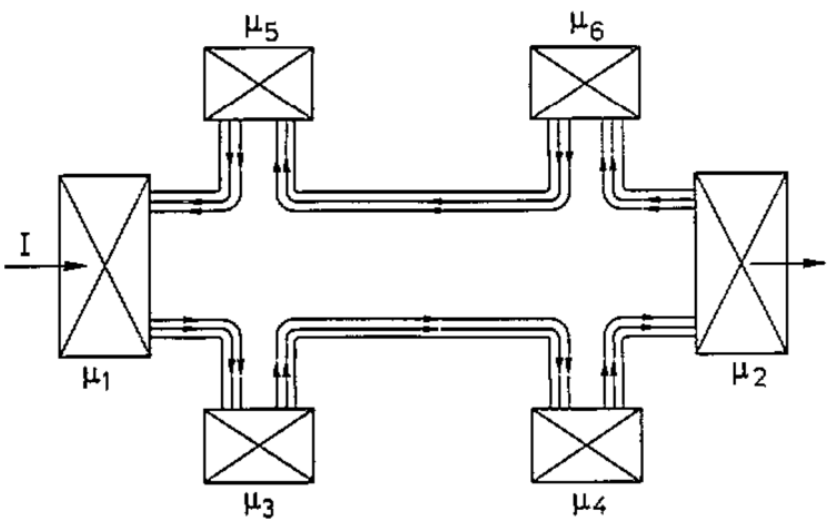


1980, von Klitzing (1985)

Source: Laboratoire national de metrologie et d'essais, French Government
Katrin Buth, Universitaet Hamburg

Laughlin explained Quantum Hall Effect using edge states (1981)





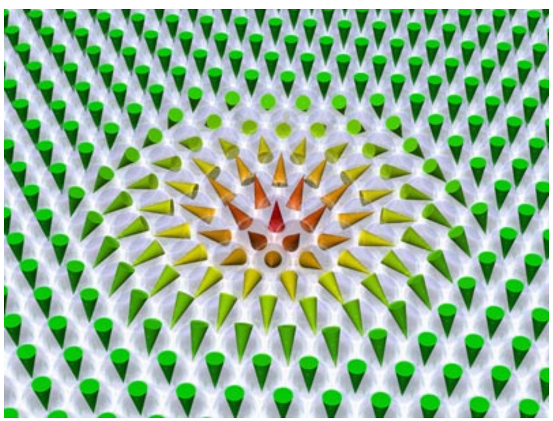
1981, Laughlin 
(1998:
– for theory of fractional Quantum Hall)

1988, Büttiker:
(Landauer picture of conductance)

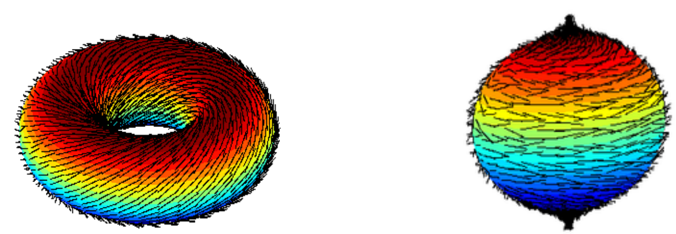
Thouless explained Quantum Hall Effect using topology (Chern number)

Calculation from Kubo formula gives for Hall conductance:

$$\sigma = \frac{e^2}{h} \sum_{n<0} \frac{1}{2\pi} \int_{BZ} d^2k \, \mathrm{Im} \, \langle \partial_{k_x} n(k) | \partial_{k_y} n(k) \rangle$$

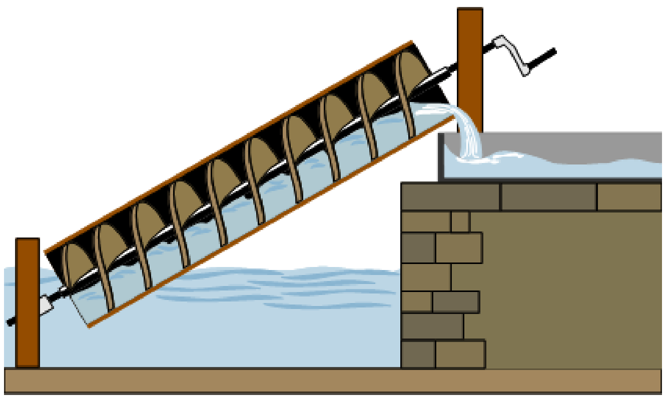


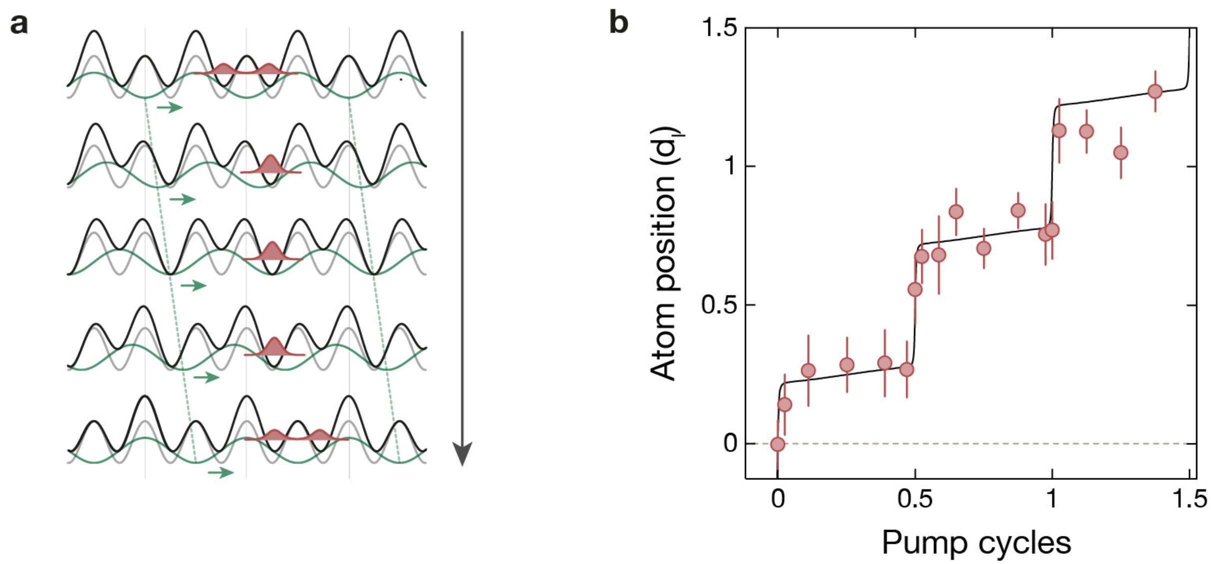
Counting skyrmions in Brillouin Zone



Thouless, Kohmoto, Nightingale, den Nijs (PRL, 1982) – TKNN

Thouless predicted Quantized Adiabatic Charge Pump in 1-dimensional quantum systems





Thouless, PRL 1983

Realized on Ultracold atoms in optical lattice,
Bloch group, ^{85}Rb (boson), MPQ Garching , 2015
Nakajima group, ^{171}Yb (fermion), Kyoto, 2015

Haldane, 1988: not magnetic field, but band topology is needed for Quantum Hall effect

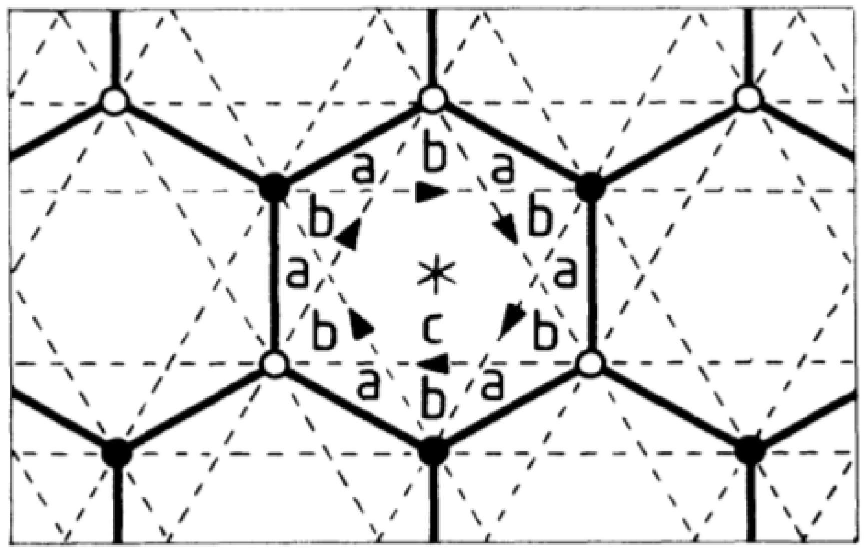
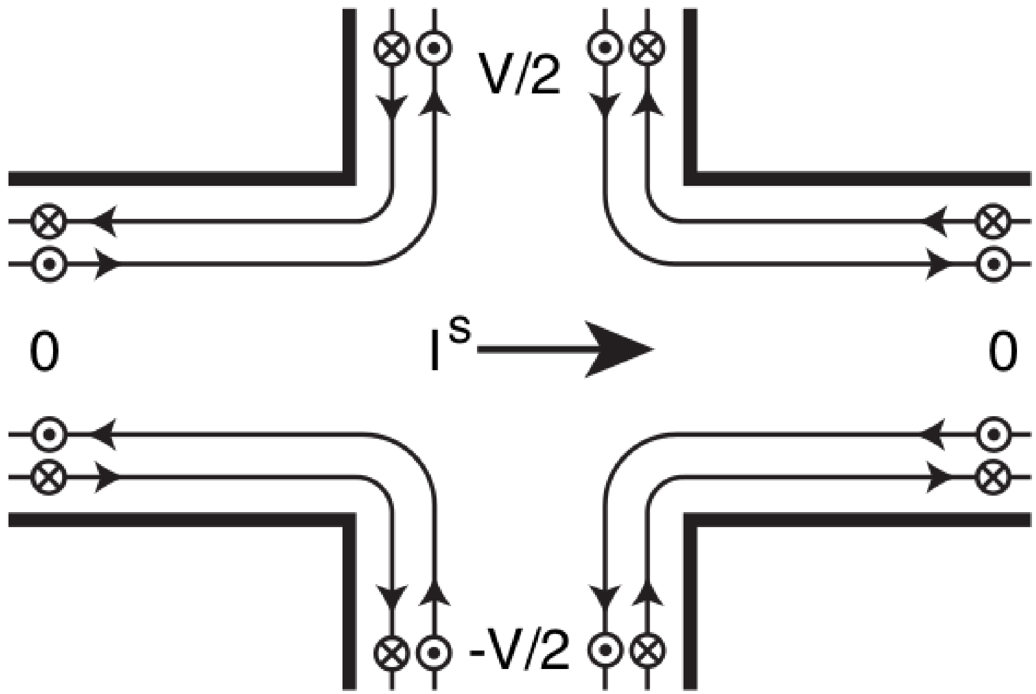


FIG. 1. The honeycomb-net model ("2D graphite")

With staggered magnetic field

Haldane (PRL, "Quantum Hall effect without Landau levels", 1988)

Kane & Mele, 2005: Graphene is 2 copies of Haldane model → wrong, but opens Topological Insulators



Predict nontrivial topology
Identify bulk topological invariant

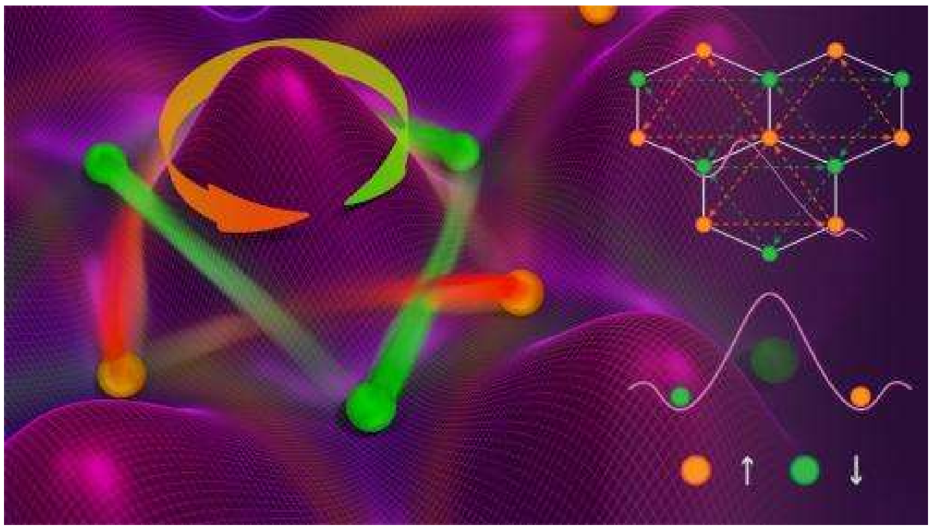
BUT: spin-orbit coupling too weak in reality

2006, Bernevig, Hughes, Zhang: HgTe

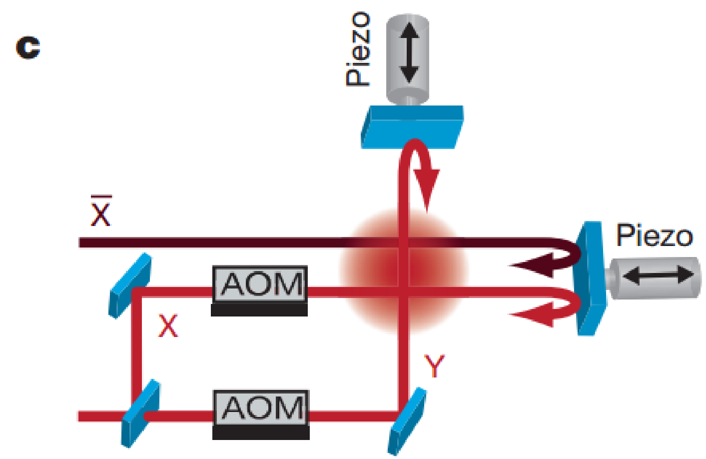
2007, Molenkamp: HgTe edge states measured

Kane & Mele (PRL, “Quantum spin Hall effect in Graphene”, 2005)

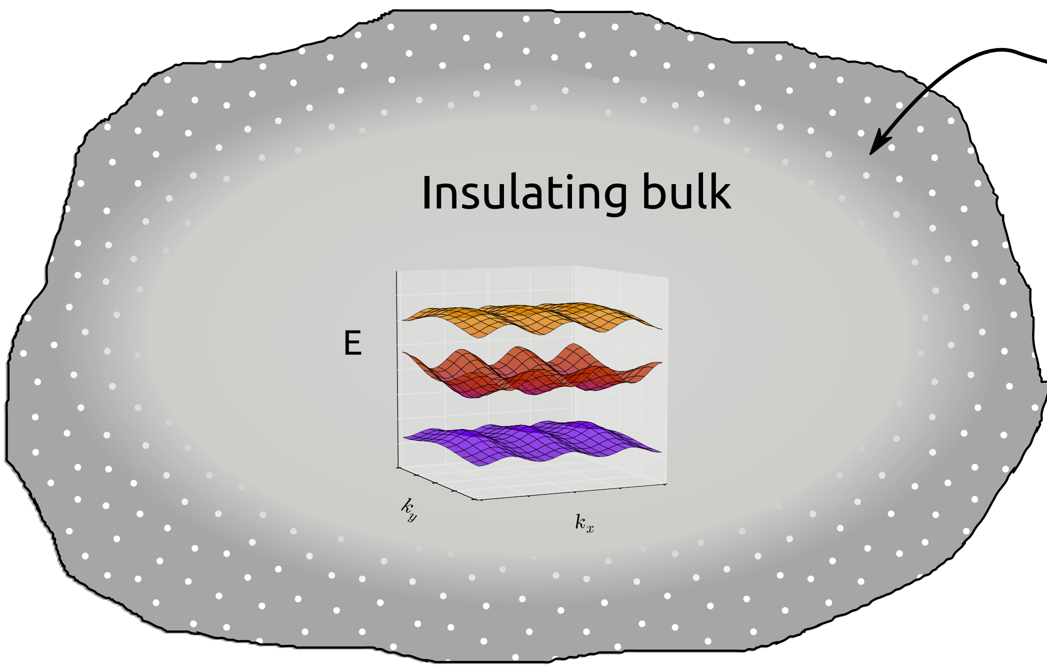
Haldane model realized in cold atomic gases in optical lattices



Esslinger group, Zurich,
ultracold fermionic ^{40}K atoms
in “shaken” optical lattice

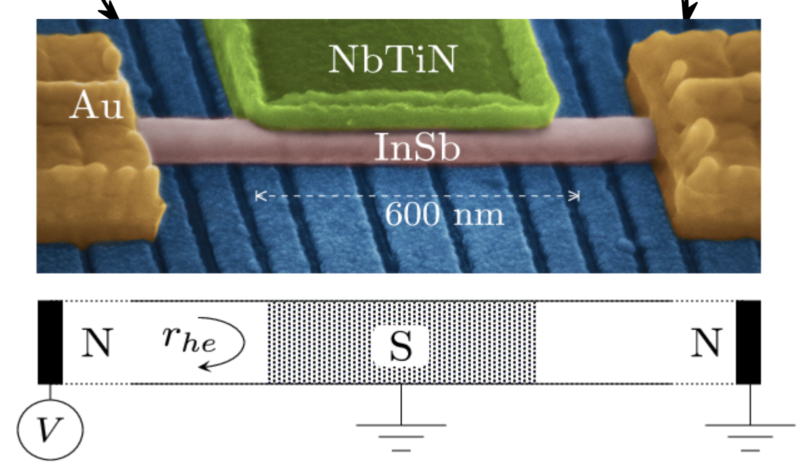


Topological Insulators: Universal, low-energy physics at the edge quantified by integers



Number of perfectly transmitting channels

Number of fixed-energy eigenstates



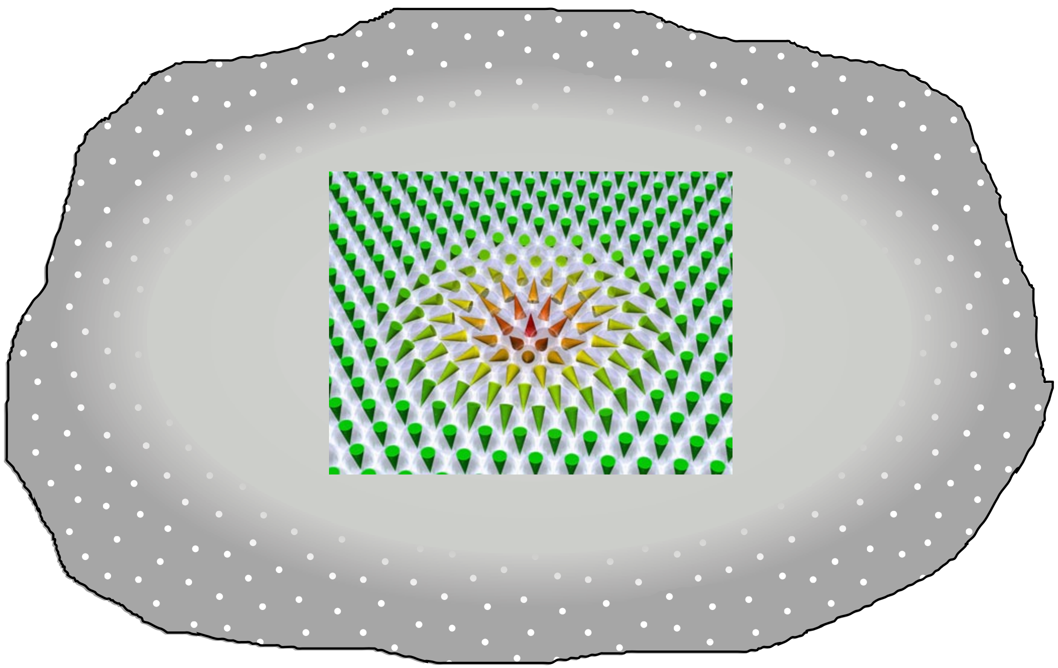
Integers quantify high-energy topology of bulk

Chern number of nth band

$$Q^{(n)} = \frac{1}{2\pi}\int_{BZ} d^2k \left(\partial_{k_x} A_y^{(n)} - \partial_{k_y} A_x^{(n)}\right)$$

$$A_\mu^{(n)}(k) = -i\langle n(k)|\partial_{k_\mu}|n(k)\rangle$$

Berry connection of nth band



Winding number of nth band

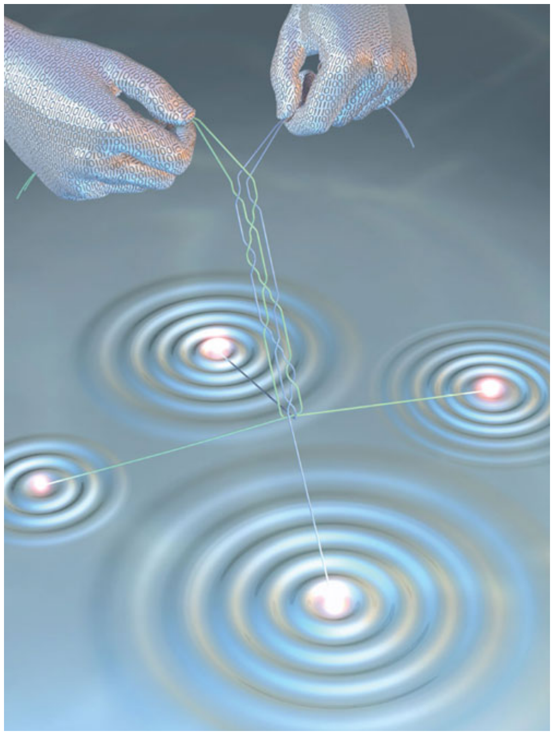
$$\nu^{(n)}(k) = \frac{-i}{2\pi}\int dk_x \langle n(k)|\hat{\Gamma}\partial_{k_\mu}|n(k)\rangle$$

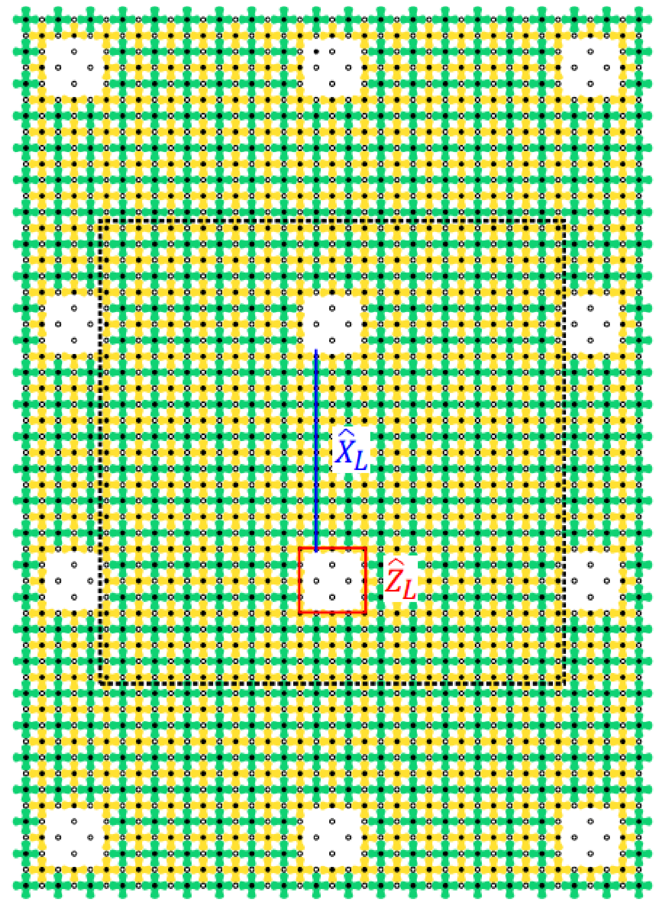
Universality classes of noninteracting topological insulators – “periodic table”

Symmetry			$\delta = d - D$							
Θ^2	Ξ^2	Π^2	0	1	2	3	4	5	6	7
0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
0	0	1	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
1	0	0	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$
0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0
−1	1	1	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$
−1	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
−1	−1	1	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
0	−1	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
1	−1	1	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$

Kitaev (2009)
Schnyder et al, NJP (2010)
Teo & Kane, PRB (2010)
Fulga et al, PRB (2012)

Fruits of Haldane's work: Quantum Computing using topological states in 2-dimensional qubit arrays



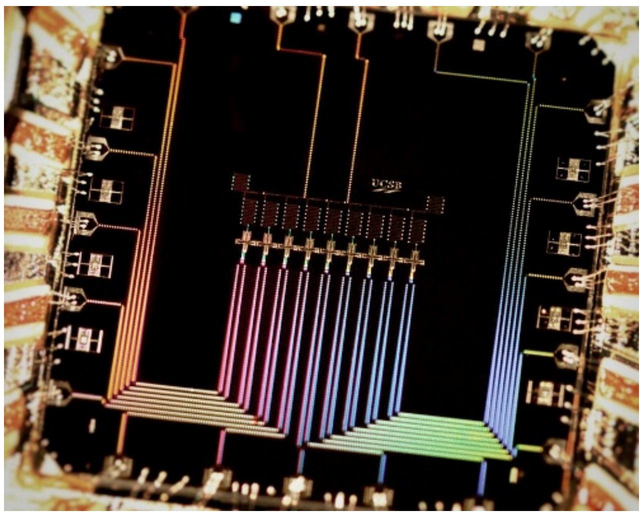


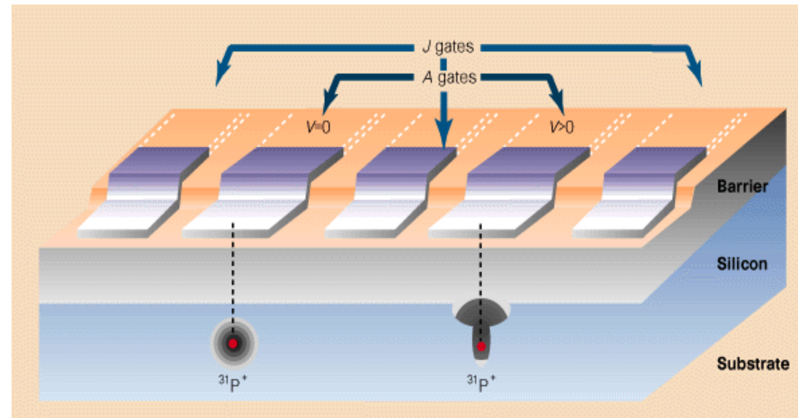
Kitaev: “Toric Code”, Ann Phys 2006
Scientific American, 2006

“Surface Code”
Fowler et al, PRA (2012)

Surface Code on superconducting integrated circuits best route to Quantum Computation

Allowed error rate 1%





Alternatives, e.g., surface code on "Kane quantum computer", Univ. Melbourne + Univ. Sidney

Martinis Group, UCSB + Google (March 2015): 9x1 qubits
Gambetta group, IBM (2015): 2x2 qubits

Europe: behind, but maybe with €1bn Quantum Technologies Flagship...

Topology in Solid State Physics

- Phases, phase transitions beyond Landau paradigm
- Robust bound states protected by nonlocality
- Promising way for quantum computing