Precesszáló kompakt kettősök szekuláris dinamikája

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Szegedi Tudományegyetem
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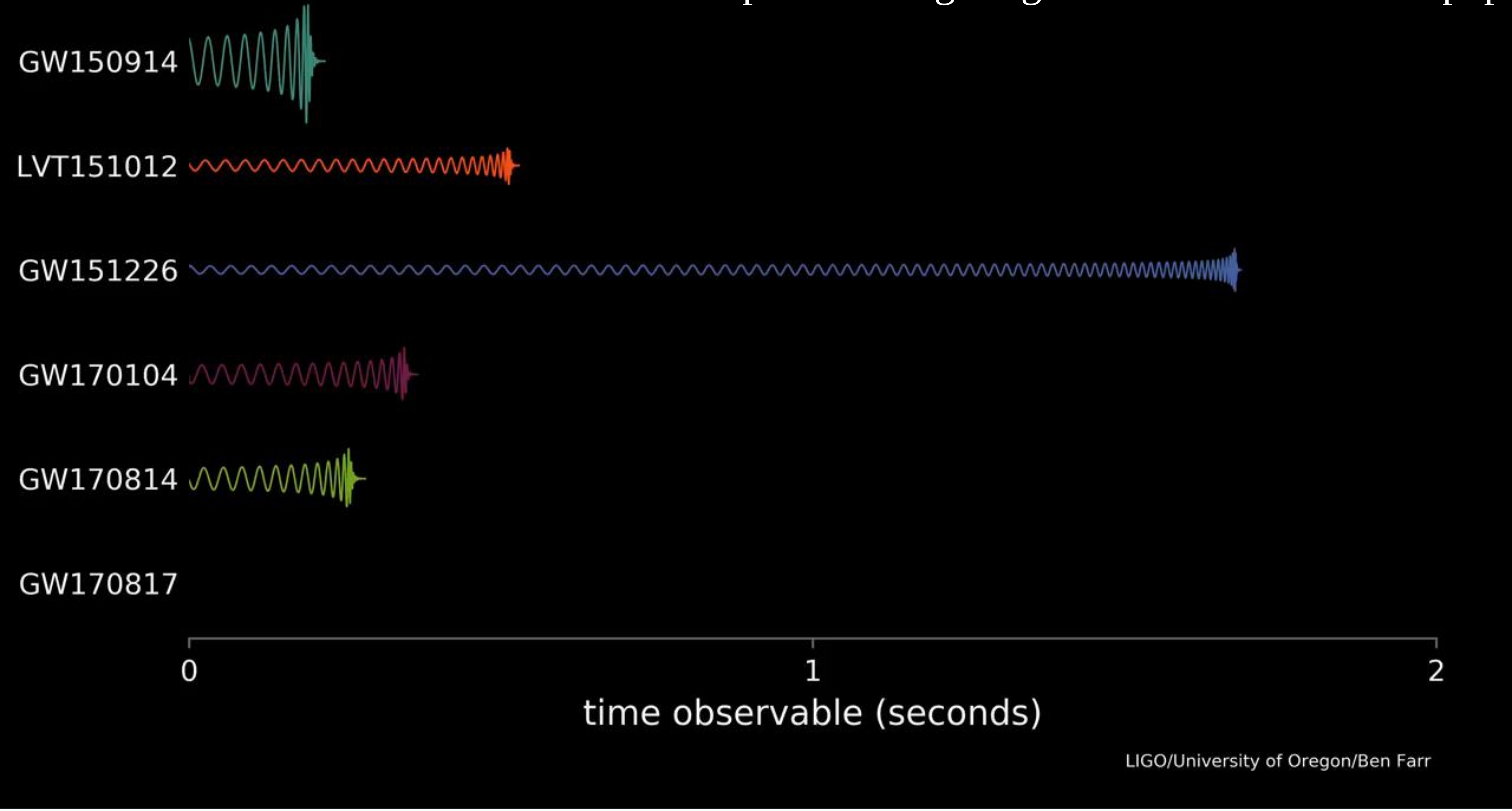
Kísérleti Fizikai Tanszék

Tartalom

- Változók a kettősök dinamikájának leírására
- Fejlődési egyenletek, első-integrálok, Kaméleon-pályák
- Szekuláris fejlődési egyenletek
- Spin szögdinamika és Spin Flip-Flop

Az észlelt gravitációs hullámok kompakt kettősökből származtak

https://www.ligo.org/detections/GW170817.php



Változók a dinamika leírására

Poszt-newtoni közelítés:

$$\varepsilon \equiv \frac{GM}{c^2 r} \approx \frac{v^2}{c^2} \ll 1$$

Pálya-alak változók:

$$\mathfrak{l}_r = \frac{cL_N}{Gm\mu}\ , \quad e_r = \frac{A_N}{Gm\mu}$$

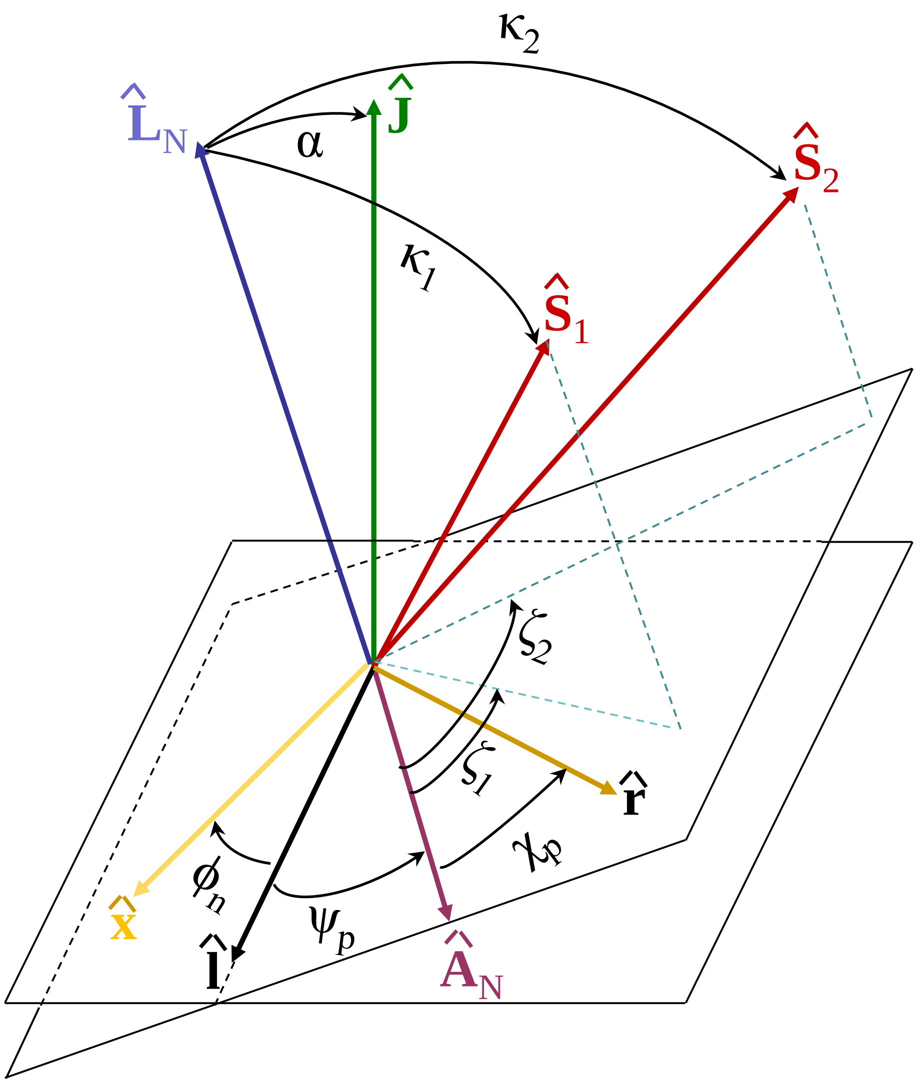
Pálya paraméterezés:

$$r = \frac{Gm}{c^2}\frac{\mathfrak{l}_r^2}{1 + e_r \cos\chi_p}$$

Konzervatív dinamika:

B. M. Barker, R. F. O'Connell, Phys. Rev. D **12**, 329 (1975).
B. M. Barker, R. F. O'Connell, Gen. Relativ. Gravit. **11**, 149 (1979).
L. E. Kidder, Phys. Rev. D **52**, 821 (1995).
LÁ Gergely, Phys.Rev. D 82, 104031 (2010).
LÁ Gergely, Phys.Rev. D 81, 084025 (2010).
E Poisson, CM Will, *Gravity: Newtonian, Post-Newtonian, Relativistic*, Cambridge Univ. Press (2014).

Szög változók:



Relatív gyorsulás

L. E. Kidder, Phys. Rev. D **52**, 821 (1995).
E Poisson, Phys. Rev. D **57**, 5287 (1998).
LÁ Gergely, Z Keresztes, Phys Rev D **67**, 024020 (2003).

G=c=1 egységekben

$$a = a_N + a_{PN} + a_{SO} + a_{SS} + a_{QM} + a_{2PN}$$

$$\mathbf{a}_\mathrm{N} = -\frac{m}{r^2}\hat{\mathbf{n}},$$

$$\mathbf{a}_\mathrm{PN} = -\frac{m}{r^2}\left\{\hat{\mathbf{n}}\left[(1+3\eta)v^2 - 2(2+\eta)\frac{m}{r} - \frac{3}{2}\eta\dot{r}^2\right] - 2(2-\eta)\dot{r}\mathbf{v}\right\},$$

$$\mathbf{a}_\mathrm{SO} = \frac{1}{r^3}\left\{6\hat{\mathbf{n}}\left[(\hat{\mathbf{n}}\times\mathbf{v})\cdot\left(2\mathbf{S} + \frac{\delta m}{m}\boldsymbol{\Delta}\right)\right] - \left[\mathbf{v}\times\left(7\mathbf{S} + 3\frac{\delta m}{m}\boldsymbol{\Delta}\right)\right] + 3\dot{r}\left[\hat{\mathbf{n}}\times\left(3\mathbf{S} + \frac{\delta m}{m}\boldsymbol{\Delta}\right)\right]\right\},$$

$$\mathbf{a}_\mathrm{SS} = -\frac{3}{\mu r^4}\{\hat{\mathbf{n}}(\mathbf{S}_1\cdot\mathbf{S}_2) + \mathbf{S}_1(\hat{\mathbf{n}}\cdot\mathbf{S}_2) + \mathbf{S}_2(\hat{\mathbf{n}}\cdot\mathbf{S}_1) - 5\hat{\mathbf{n}}(\hat{\mathbf{n}}\cdot\mathbf{S}_1)(\hat{\mathbf{n}}\cdot\mathbf{S}_2)\},$$

$$\mathbf{a}_{QM} = -\frac{3}{2}\frac{m^3}{r^7}\sum_{i=1}^{2} p_i\left[5\left(\hat{\mathbf{S}}_i\cdot\mathbf{r}\right)^2 - r^2\right]\mathbf{r} + \frac{3}{}\frac{m^3}{r^5}\sum_{i=1}^{2} p_i\left(\hat{\mathbf{S}}_i\cdot\mathbf{r}\right)\hat{\mathbf{S}}_i$$

$$\begin{aligned}\mathbf{a}_\mathrm{2PN} = -\frac{m}{r^2}\Big\{\hat{\mathbf{n}}\Big[&\frac{3}{4}(12+29\eta)\left(\frac{m}{r}\right)^2 + \eta(3-4\eta)v^4 + \frac{15}{8}\eta(1-3\eta)\dot{r}^4\\ &- \frac{3}{2}\eta(3-4\eta)v^2\dot{r}^2 - \frac{1}{2}\eta(13-4\eta)\frac{m}{r}v^2 - (2+25\eta+2\eta^2)\frac{m}{r}\dot{r}^2\Big]\\ &- \frac{1}{2}\dot{r}\mathbf{v}\left[\eta(15+4\eta)v^2 - (4+41\eta+8\eta^2)\frac{m}{r} - 3\eta(3+2\eta)\dot{r}^2\right]\Big\},\end{aligned}$$

Spin fejlődési egyenletek

L. E. Kidder, Phys. Rev. D **52**, 821 (1995).
E Poisson, Phys. Rev. D **57**, 5287 (1998).
LÁ Gergely, Z Keresztes, Phys Rev D **67**, 024020 (2003).

G=c=1 egységekben

$$\dot{\mathbf{S}}_i = (\mathbf{P}_{SO} + \mathbf{P}_{SS} + \mathbf{P}_{QM}) \times \mathbf{S}_i$$

$$\mathbf{P}_{SO} = \frac{1}{r^3}\left(2 + \frac{3m_j}{2m_i}\right)\mathbf{L}_N$$

i,j=1,2
$i \neq j$

$$\mathbf{P}_{SS} = \frac{1}{r^3}\left[-\mathbf{S}_j + 3\left(\mathbf{n}\cdot\mathbf{S}_j\right)\mathbf{n}\right]$$

$$\mathbf{P}_{QM} = -\frac{3\mu m^3}{r^3} p_i \left(\mathbf{n}\cdot\mathbf{S}_i\right)\mathbf{n}$$

Fejlődési egyenlet az excentricitásra

LÁ Gergely, Z Keresztes, PRD **91**, 024012 (2015)

$$\dot{e}_r = \frac{\mathfrak{l}_r}{1+e_r\cos\chi_p}\Big[-\left(\mathfrak{a}\cdot\hat{\mathbf{A}}_{\mathbf{N}}\right)(e_r+\cos\chi_p)\sin\chi_p + \left(\mathfrak{a}\cdot\hat{\mathbf{Q}}_{\mathbf{N}}\right)\left(1+2e_r\cos\chi_p+\cos^2\chi_p\right)\Big] ,$$

$$\mathfrak{a}\cdot\hat{\mathbf{Q}}_{\mathbf{N}} = \mathfrak{a}_2^{PN}+\mathfrak{a}_2^{2PN}+\mathfrak{a}_2^{SO}+\mathfrak{a}_2^{SS}+\mathfrak{a}_2^{QM} ,$$

$$\mathfrak{a}_2^{PN} = \frac{(1+e_r\cos\chi_p)^2\sin\chi_p}{\mathfrak{l}_r^6}\sum_{k=0}^{2}\mathfrak{c}_{2(k)}^{PN}\cos^k\chi_p ,$$

$$\mathfrak{a}_2^{2PN} = \frac{(1+e_r\cos\chi_p)^2\sin\chi_p}{\mathfrak{l}_r^8}\sum_{k=0}^{4}\mathfrak{c}_{2(k)}^{2PN}\cos^k\chi_p ,$$

$$\mathfrak{a}_2^{SO} = \frac{\eta(1+e_r\cos\chi_p)^3\sin\chi_p}{2\mathfrak{l}_r^7}$$

$$\mathfrak{a}_2^{SS} = \frac{3\eta(1+e_r\cos\chi_p)^4}{\mathfrak{l}_r^8}\chi_1\chi_2\Big\{-\cos\kappa_1\cos\kappa_2 \times\sin\chi_p+\frac{1}{4}\sin\kappa_1\sin\kappa_2\left[2\cos\zeta_{(-)}\sin\chi_p - \sin\left(\chi_p-\zeta_{(+)}\right)+5\sin\left(3\chi_p-\zeta_{(+)}\right)\right]\Big\} ,$$

$$\mathfrak{a}_2^{QM} = -\frac{3\eta(1+e_r\cos\chi_p)^4}{2\mathfrak{l}_r^8}\sum_{k=1}^{2}w_k\nu^{2k-3}\chi_k^2\Big\{\sin\chi_p -\frac{1}{2}\sin^2\kappa_k\left[\sin\zeta_k+5\sin(2\chi_p-\zeta_k)\right] \times\cos(\chi_p-\zeta_k)\Big\} ,$$

$$\mathfrak{a}\cdot\hat{\mathbf{A}}_{\mathbf{N}} = \mathfrak{a}_1^{PN}+\mathfrak{a}_1^{2PN}+\mathfrak{a}_1^{SO}+\mathfrak{a}_1^{SS}+\mathfrak{a}_1^{QM} ,$$

$$\mathfrak{a}_1^{PN} = \frac{(1+e_r\cos\chi_p)^2}{\mathfrak{l}_r^6}\sum_{k=0}^{3}\mathfrak{c}_{1(k)}^{PN}\cos^k\chi_p ,$$

$$\mathfrak{a}_1^{2PN} = \frac{(1+e_r\cos\chi_p)^2}{\mathfrak{l}_r^8}\sum_{k=0}^{5}\mathfrak{c}_{1(k)}^{2PN}\cos^k\chi_p ,$$

$$\mathfrak{a}_1^{SO} = \frac{\eta(1+e_r\cos\chi_p)^3(e_r+\cos\chi_p)}{2\mathfrak{l}_r^7} \times\sum_{k=1}^{2}\left(4\nu^{2k-3}+3\right)\chi_k\cos\kappa_k ,$$

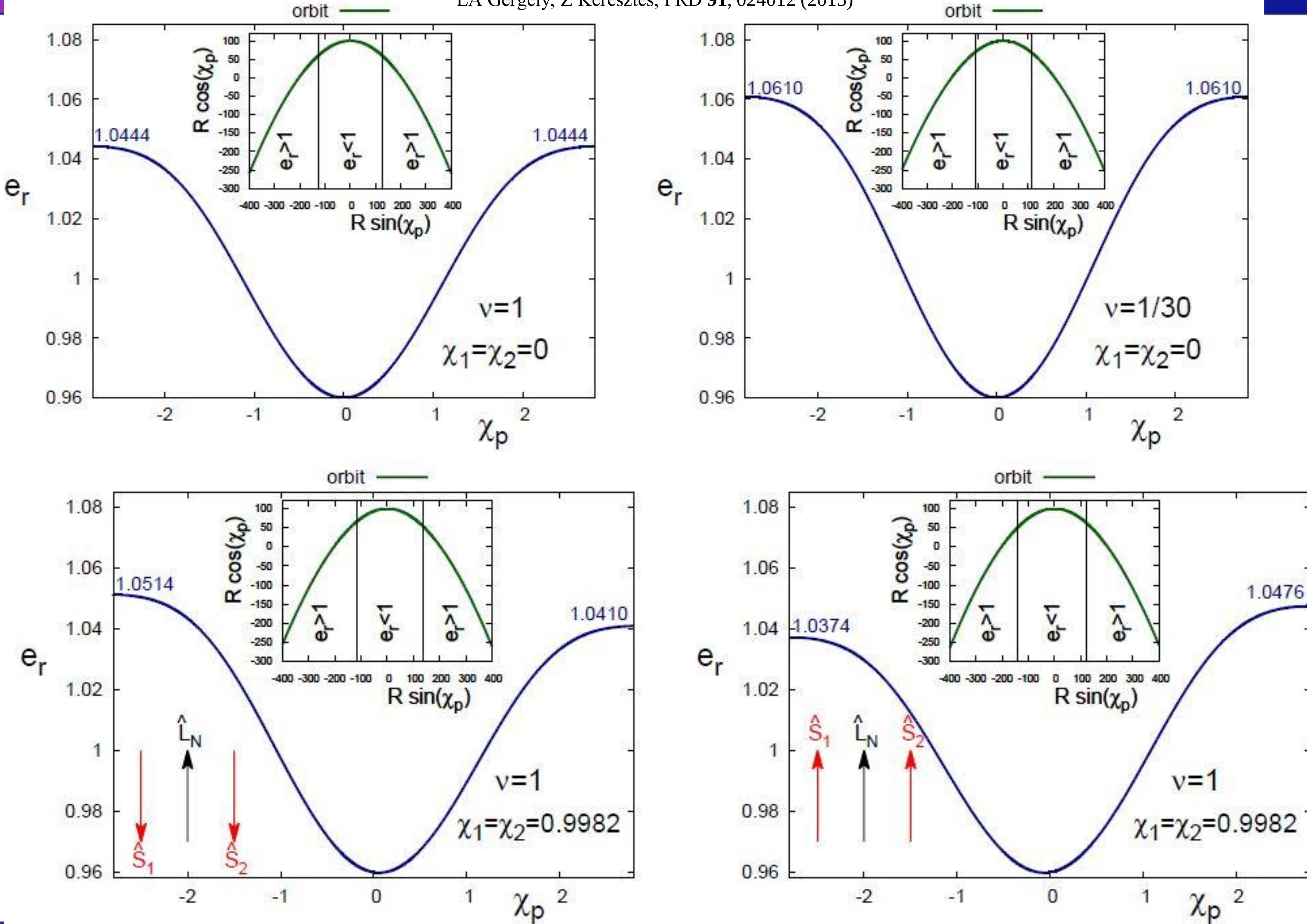
$$\mathfrak{a}_1^{SS} = \frac{3\eta(1+e_r\cos\chi_p)^4}{\mathfrak{l}_r^8}\chi_1\chi_2\Big\{-\cos\kappa_1\cos\kappa_2 \times\cos\chi_p+\frac{1}{4}\sin\kappa_1\sin\kappa_2\left[2\cos\zeta_{(-)}\cos\chi_p + \cos\left(\chi_p-\zeta_{(+)}\right)+5\cos\left(3\chi_p-\zeta_{(+)}\right)\right]\Big\} ,$$

$$\mathfrak{a}_1^{QM} = -\frac{3\eta(1+e_r\cos\chi_p)^4}{2\mathfrak{l}_r^8}\sum_{k=1}^{2}w_k\nu^{2k-3}\chi_k^2\Big\{\cos\chi_p -\frac{1}{2}\sin^2\kappa_k\left[\cos\zeta_k+5\cos(2\chi_p-\zeta_k)\right] \times\cos(\chi_p-\zeta_k)\Big\} ,$$

$R = c^2 r/Gm$

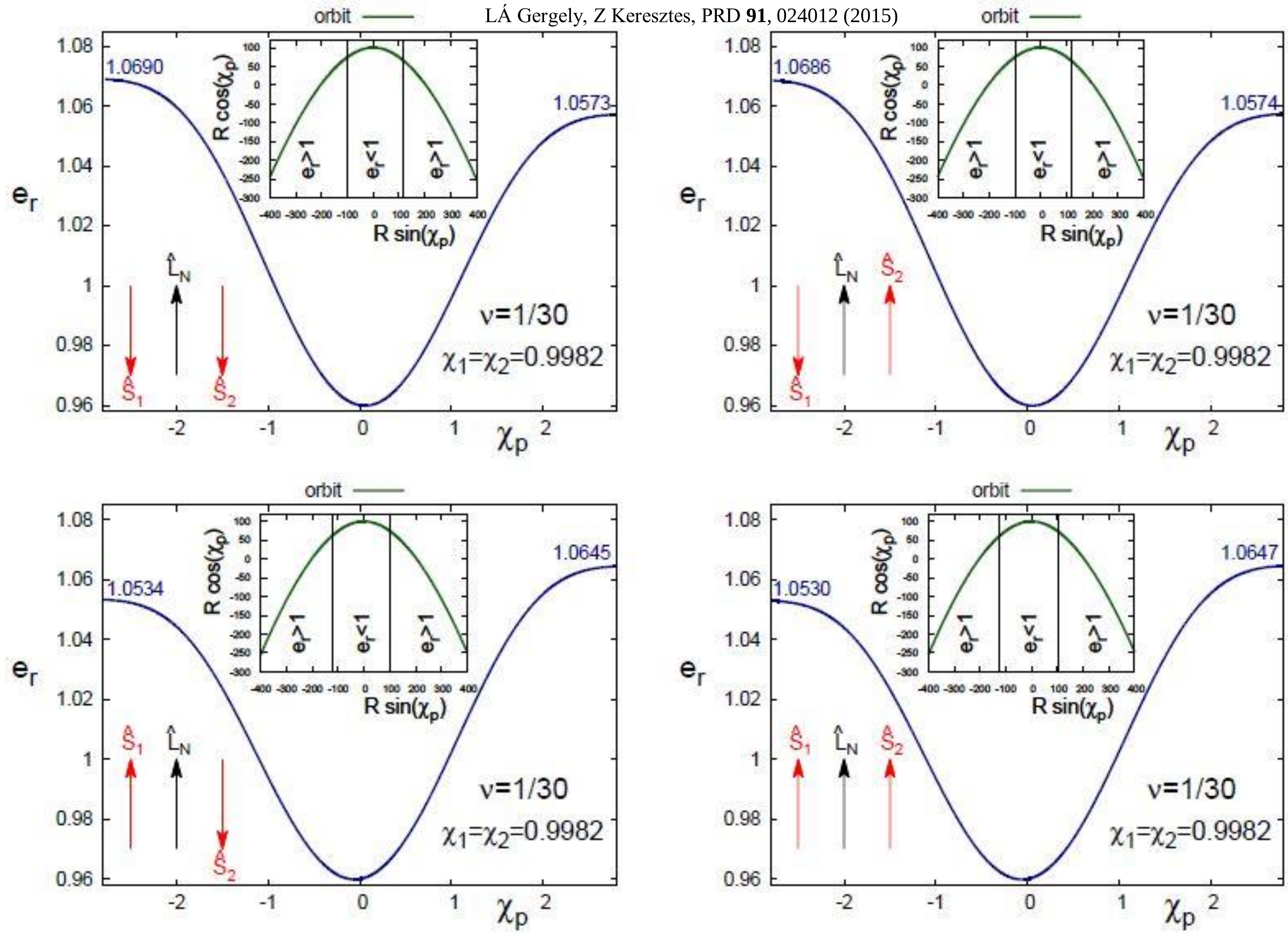
Kaméleon pályák

LÁ Gergely, Z Keresztes, PRD **91**, 024012 (2015)



Kaméleon pályák

LÁ Gergely, Z Keresztes, PRD **91**, 024012 (2015)



Első integrálok

Energia megmaradás:

$$\mathfrak{E} = \mathfrak{E}_N + \mathfrak{E}_{PN} + \mathfrak{E}_{2PN} + \mathfrak{E}_{SS} + \mathfrak{E}_{QM} \ ,$$

$$\mathfrak{E}_N = \frac{e_r^2 - 1}{2\mathfrak{l}_r^2} \ ,$$

$$\mathfrak{E}_{PN} = \frac{1}{8\mathfrak{l}_r^4} \sum_{k=0}^{3} \mathfrak{q}_k e_r^k \cos^k \chi_p \ ,$$

$$\mathfrak{E}_{2PN} = \frac{1}{16\mathfrak{l}_r^6} \sum_{k=0}^{5} \mathfrak{s}_k e_r^k \cos^k \chi_p \ ,$$

$$\mathfrak{E}_{SS} = -\frac{\eta\left(1 + e_r \cos\chi_p\right)^3}{2\mathfrak{l}_r^6} \chi_1 \chi_2 \times \Big\{ 2\cos\kappa_1 \cos\kappa_2 - \sin\kappa_1 \sin\kappa_2 \times \left[\cos\zeta_{(-)} + 3\cos\left(2\chi_p - \zeta_{(+)}\right)\right] \Big\} \ ,$$

$$\mathfrak{E}_{QM} = -\frac{\eta\left(1 + e_r \cos\chi_p\right)^3}{2\mathfrak{l}_r^6} \sum_{i=1}^{2} w_i \chi_i^2 \nu^{2i-3} \times \left[1 - 3\sin^2\kappa_i \cos^2\left(\chi_p - \zeta_i\right)\right] \ .$$

Teljes impulzusmomentum megmaradás:

$$0 = \sum_{i=1}^{2} \chi_i \sin\kappa_i \Big[\nu^{2i-3} \cos\left(\zeta_i + \psi_p\right) - \frac{\eta\left(4\nu^{2i-3} + 3\right)\left(1 + e_r \cos\chi_p\right)}{2\mathfrak{l}_r^2} \times \sin\left(\chi_p + \psi_p\right) \sin\left(\chi_p - \zeta_i\right) \Big] \ ,$$

$$\mathfrak{J} \sin\alpha = \sum_{i=1}^{2} \chi_i \sin\kappa_i \Big[\nu^{2i-3} \sin\left(\zeta_i + \psi_p\right) + \frac{\eta\left(4\nu^{2i-3} + 3\right)\left(1 + e_r \cos\chi_p\right)}{2\mathfrak{l}_r^2} \times \cos\left(\chi_p + \psi_p\right) \sin\left(\chi_p - \zeta_i\right) \Big] \ ,$$

$$\mathfrak{J} \cos\alpha = \mathfrak{l}_r \left(1 + \epsilon_{PN} + \epsilon_{2PN}\right) + \sum_{i=1}^{2} \chi_i \cos\kappa_i \times \left[\nu^{2i-3} - \frac{\eta\left(4\nu^{2i-3} + 3\right)\left(1 + e_r \cos\chi_p\right)}{2\mathfrak{l}_r^2} \right] \ ,$$

LÁ Gergely, Z Keresztes, PRD **91**, 024012 (2015)

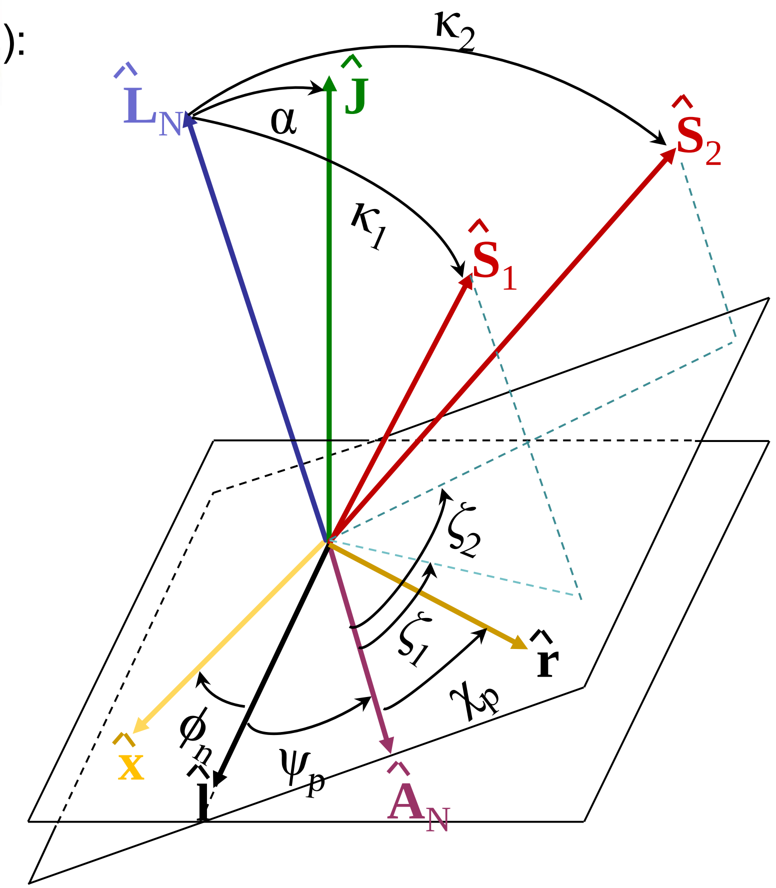
Időskálák; szekuláris fejlődési egyenletek

Három időskála dimenziómentes egységekben ($\tau = \frac{c^3 t}{Gm}$):

$$\tau_{orb} \approx 2\pi\varepsilon^{-3/2}$$

$$\tau_{prec} \approx \pi\varepsilon^{-5/2}/\eta$$

$$\tau_{GW} \approx 5\varepsilon^{-4}/(32\eta)$$



Szekuláris differenciálegyenletek:

$$T\bar{f} = \int_0^T f(t)\,dt = \int_0^{2\pi} f(\chi_p)\frac{1}{\dot{\chi}_p}d\chi_p$$

$f = \dot{Y}$ ahol

$$Y = \{\iota_r, e_r, \alpha, \phi_n, \psi_p, \kappa_i, \zeta_i\}$$

Példák:

$$\bar{\dot{e}}_r = \bar{\dot{e}}_r^{PN} = \bar{\dot{e}}_r^{SO} = \bar{\dot{e}}_r^{SS} = \bar{\dot{e}}_r^{QM} = \bar{\dot{e}}_r^{2PN} = 0$$

$$\bar{\dot{\iota}}_r = \bar{\dot{\iota}}_r^{PN} = \bar{\dot{\iota}}_r^{SO} = \bar{\dot{\iota}}_r^{SS} = \bar{\dot{\iota}}_r^{QM} = \bar{\dot{\iota}}_r^{2PN} = 0$$

Kényszerek: $\sum_{i=1}^{2} \nu^{2i-3}\chi_i \sin\kappa_i \cos(\zeta_i + \psi_p) = 0$

$$\bar{\iota}_r = \sum_{i=1}^{2} \nu^{2i-3}\chi_i\left[\sin\kappa_i \sin(\zeta_i + \psi_p)\cot\alpha - \cos\kappa_i\right]$$

Szekuláris és pillanatnyi evolúciók

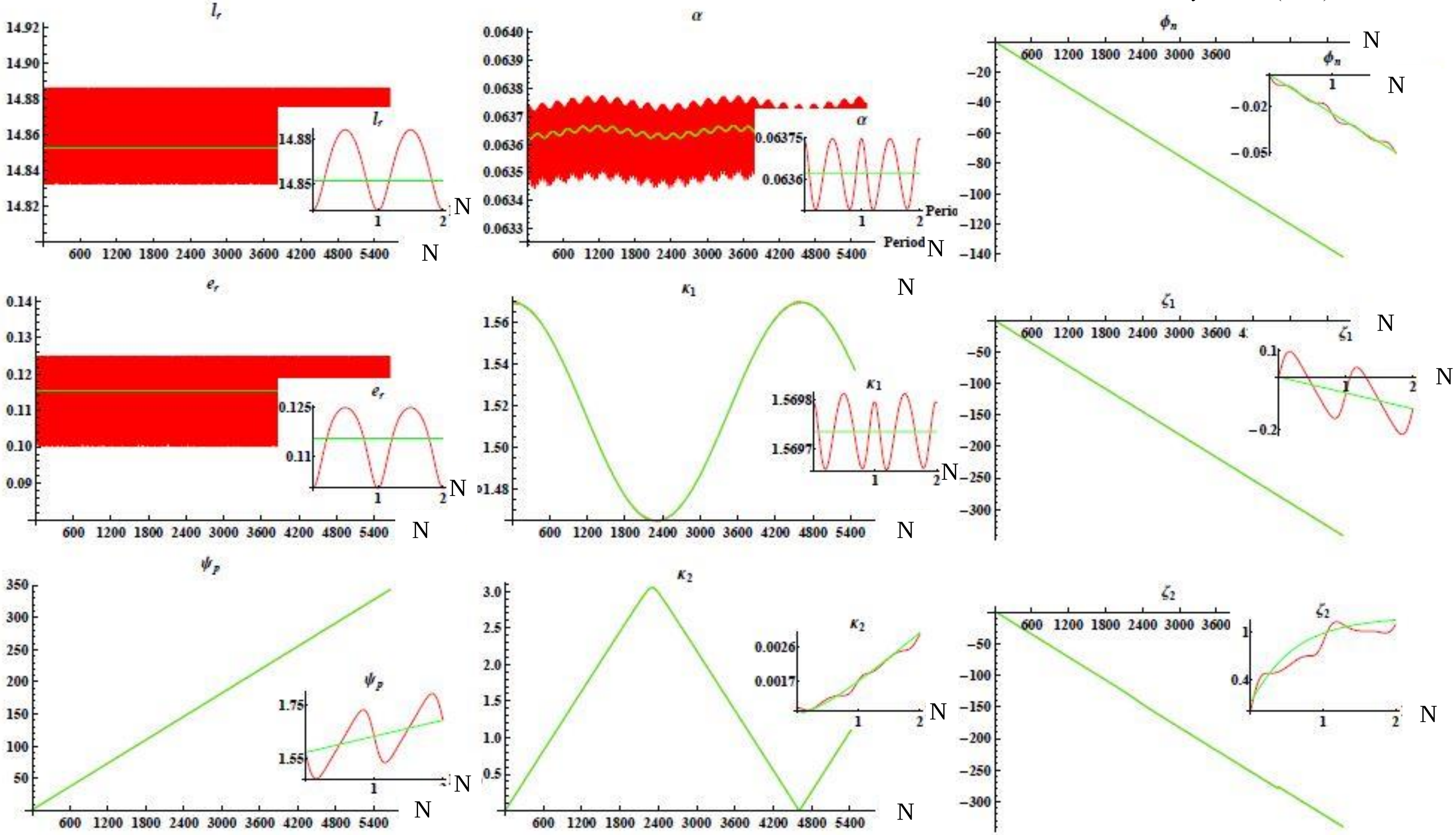
A pályaperiódus maximális száma, amikor a 2.5 PN rendű gravitációs sugárzási visszahatás elhanyagolása miatt nagyjából p%-os hibát vétünk:

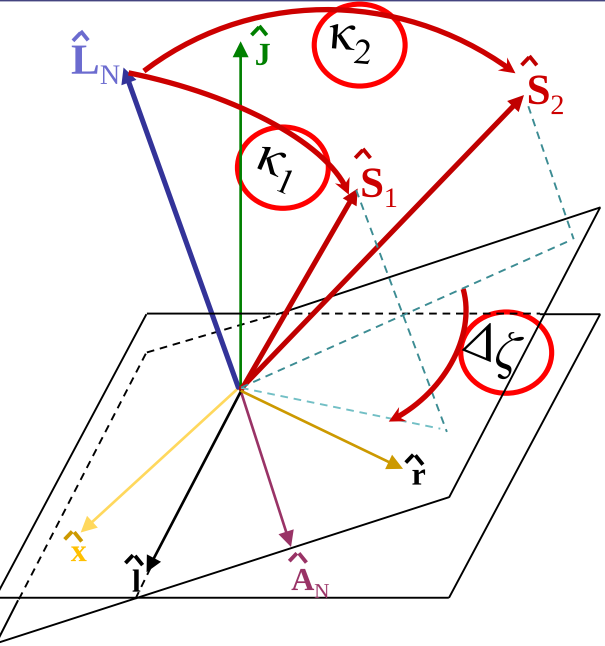
$$N_{\max} = p\varepsilon^{-5/2}/100$$

$$N_{\max}\tau_{orb} \approx 0.1p\tau_{GW}$$

Szögek radiánban értendők.

LÁ Gergely, Z Keresztes, M Tápai, Publikálásra elfogadva a *Universe* folyóiratban (2018)





Kis dimenziómentes spinparaméter arány.

$$\chi_2 \ll \chi_1: \qquad \frac{d(\Delta\zeta)}{d\bar{\mathfrak{t}}} \approx A_{S_1} + B_{S_1}\cot\kappa_2\cos(\Delta\zeta) \ ,$$

$$\chi_i = \frac{cS_i}{Gm_i^2} \qquad \frac{d\kappa_2}{d\bar{\mathfrak{t}}} \approx B_{S_1}\sin(\Delta\zeta) \ , \qquad \frac{d\kappa_1}{d\bar{\mathfrak{t}}} \approx 0 \ .$$

Kvázi konstans együtthatók:

$$A_{S_1} = \frac{3\eta\pi}{\mathfrak{T}\,\mathfrak{l}_r^2}\left(\nu - \frac{1}{\nu}\right) + \frac{3\eta\pi\chi_1}{\mathfrak{T}\,\mathfrak{l}_r^3}\left(1 + \frac{2}{\nu} - w_1 - \frac{w_1\chi_1}{\nu\mathfrak{l}_r}\cos\kappa_1\right)\cos\kappa_1 \ ,$$

$$B_{S_1} = -\frac{3\eta\pi\chi_1}{\mathfrak{T}\,\mathfrak{l}_r^3}\left(1 + \frac{1}{\nu} - \frac{w_1\chi_1}{\nu\mathfrak{l}_r}\cos\kappa_1\right)\sin\kappa_1 \ . \qquad \nu = \frac{m_2}{m_1} \ ,$$

$$\eta = \frac{m_1 m_2}{(m_1+m_2)^2} \ .$$

$\Downarrow$

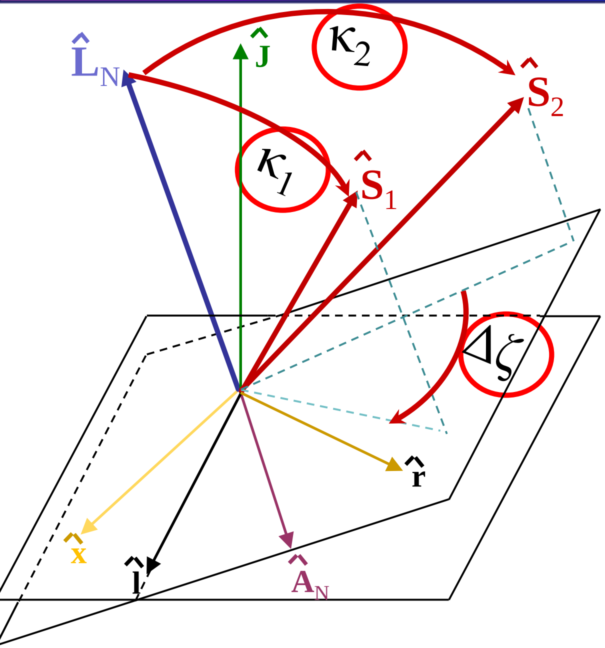
Megoldás:

$$\cos\kappa_2 = A_{S_1}C + \epsilon\frac{B_{S_1}}{\Omega_{S_1}}\sqrt{1-\Omega_{S_1}^2 C^2}\sin\left(\Omega_{S_1}\bar{\mathfrak{t}} + D\right) \ ,$$

$$\sin\kappa_2\sin\Delta\zeta = -\epsilon\sqrt{1-\Omega_{S_1}^2 C^2}\cos\left(\Omega_{S_1}\bar{\mathfrak{t}} + D\right) \ .$$

ahol $\Omega_{S_1} = \sqrt{A_{S_1}^2 + B_{S_1}^2}$, C , D integrációs konstansok, $\epsilon = \pm 1$.

Spin szögdinamika



Megoldás:

$$\cos\kappa_2 = A_{S_1}C + \epsilon\frac{B_{S_1}}{\Omega_{S_1}}\sqrt{1-\Omega_{S_1}^2 C^2}\sin\left(\Omega_{S_1}\bar{\mathfrak{t}} + D\right) \;,$$

$$\sin\kappa_2 \sin\Delta\zeta = -\epsilon\sqrt{1-\Omega_{S_1}^2 C^2}\cos\left(\Omega_{S_1}\bar{\mathfrak{t}} + D\right) \;.$$

Kvázi konstans együtthatók: $\Omega_{S_1} = \sqrt{A_{S_1}^2 + B_{S_1}^2}\;,$

$$A_{S_1} = \frac{3\eta\pi}{\mathfrak{T}\,\bar{\mathfrak{l}}_r^2}\left(\nu - \frac{1}{\nu}\right) + \frac{3\eta\pi\chi_1}{\mathfrak{T}\,\bar{\mathfrak{l}}_r^3}\left(1 + \frac{2}{\nu} - w_1 - \frac{w_1\chi_1}{\nu\bar{\mathfrak{l}}_r}\cos\kappa_1\right)\cos\kappa_1 \;,$$

$$B_{S_1} = -\frac{3\eta\pi\chi_1}{\mathfrak{T}\,\bar{\mathfrak{l}}_r^3}\left(1 + \frac{1}{\nu} - \frac{w_1\chi_1}{\nu\bar{\mathfrak{l}}_r}\cos\kappa_1\right)\sin\kappa_1 \;.$$

Kisebb spinvektor áthalad az $\mathbf{L}_{\mathrm{N}}$ által meghatározott irányon:

$$C = \epsilon_{\cos\kappa_2}\frac{A_{S_1}}{\Omega_{S_1}^2}$$

$$\Delta\zeta \rightarrow \epsilon\frac{\pi}{2} \;\text{as}\; \kappa_2 \rightarrow \{0,\pi\}$$

Kisebb spin paraméterű spin polárszög koszinuszának megváltozása:

$$|\Delta\cos\kappa_2| = \sqrt{\frac{B_{S_1}^2}{A_{S_1}^2 + B_{S_1}^2} - B_{S_1}^2 C^2}$$

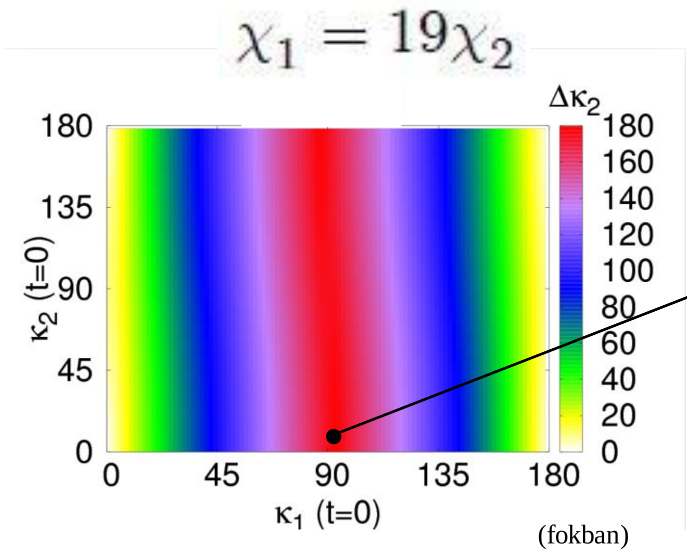
Nagy Flip-Flop: $A_{S_1}^2 \ll B_{S_1}^2 \;\Longrightarrow\; \nu \approx 1$

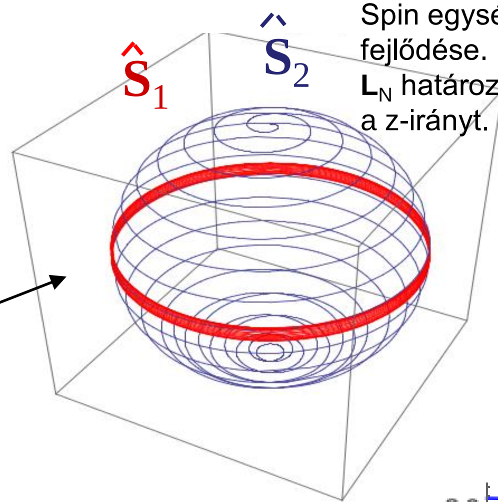
1) $\cos\kappa_1 \approx 0$

Kvázi körpályára: CO Lousto, J Healy, Phys. Rev Lett. **114**, 141101 (2015)

2) $w_1 \neq 1$ neutron csillagok ?

Standard kvázi körpályás eredmények reprodukálva kis excentricitású pályákra





Spin egységvektorok fejlődése.
$\mathbf{L}_N$ határozza meg a z-irányt.

Kvázi körpálya:

CO Lousto, J Healy, Phys. Rev Lett. **114**, 141101 (2015)

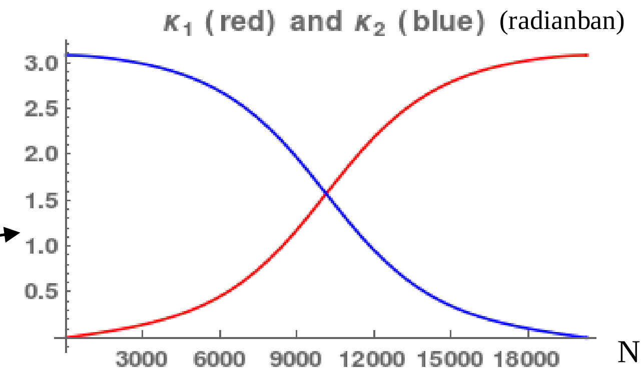
D Gerosa, M Kesden, R O'Shaughnessy, A Klein, E Berti, U Sperhake, D Trifirn, Phys. Rev. Lett. **115**, 141102 (2015)

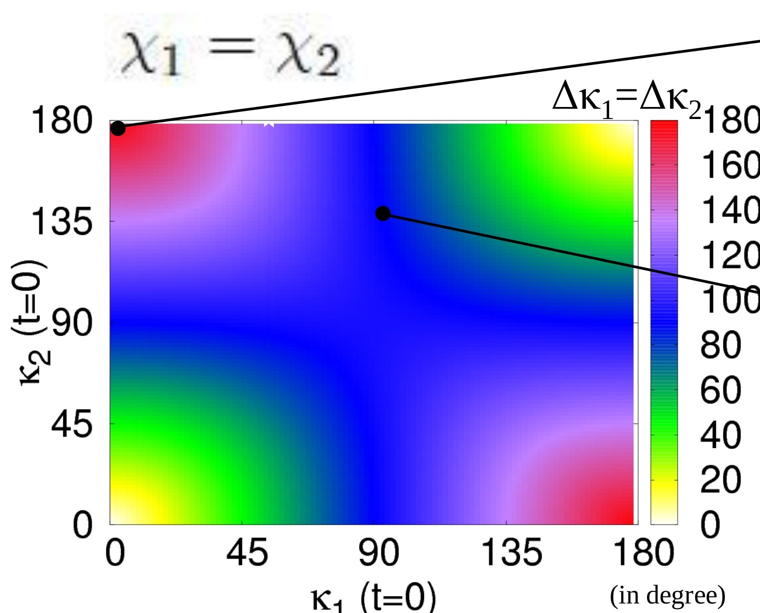
CO Lousto, J Healy, H Nakano, Phys. Rev D **93**, 044031 (2016)

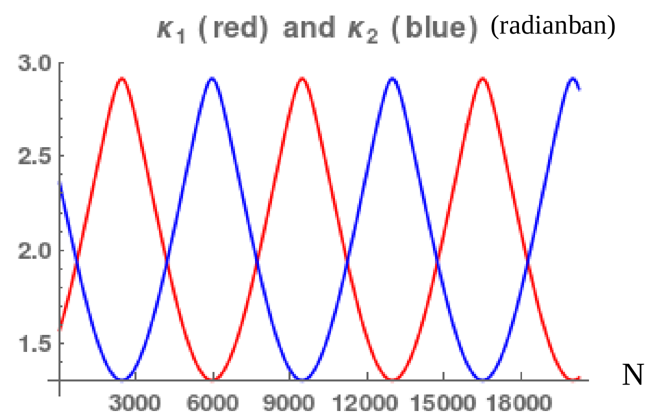
CO Lousto, J Healy, Phys. Rev D **93**, 124074 (2016)

e_r=0.1,

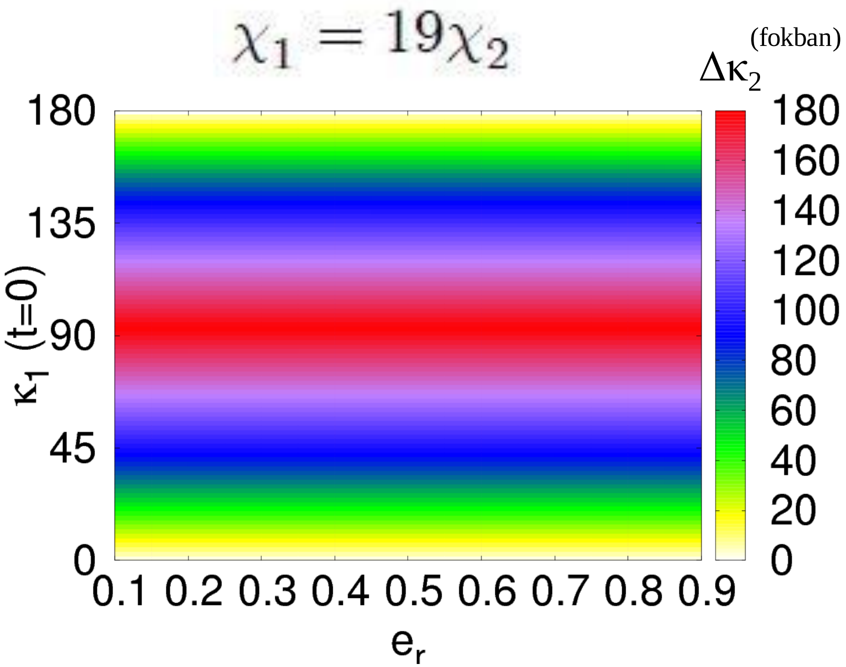
ν=1



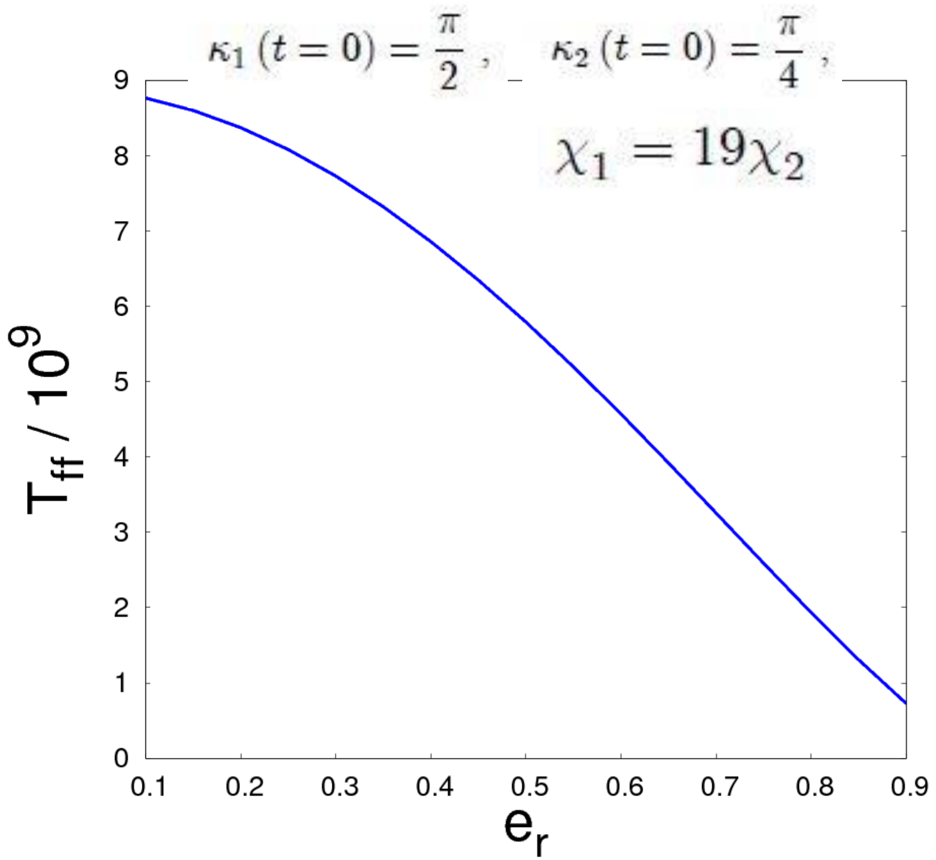


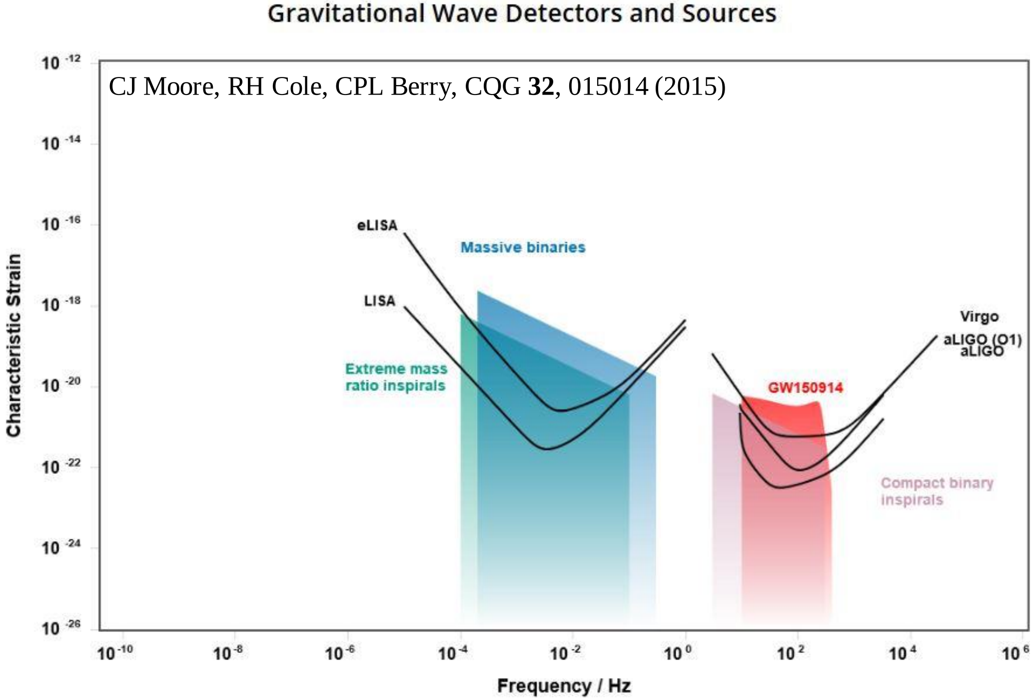


Excentricitás hatása

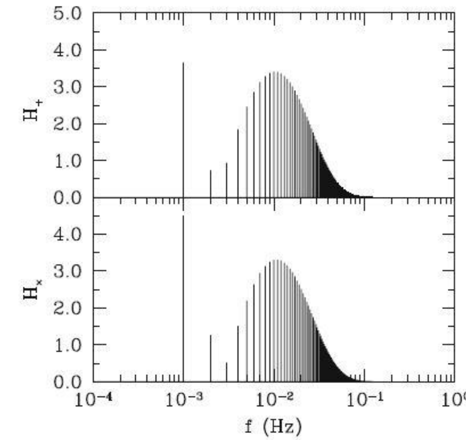


Flip-Flop effektus periódusideje dimenziótlan egységekben.





Excentricitás hatása a GH spektrumra



G Aznar-Siguán, E García-Berro, P Lorén-Aguilar, The Gravitational Wave Emission of White Dwarf Dynamical Interactions in Gravitational Wave Astrophysics ed. by CF Sopuerta (2015)

Fig. 1 Gravitational waveforms for a close encounter in which an eccentric binary is formed. For this particular case both white dwarfs have equal masses, $M = 0.8\,M_\odot$, whilst the adopted initial conditions lead to the formation of a binary system with eccentricity $e = 0.820$ and frequency $f = 0.001$ Hz. In the *top panels* we show the h_+ and $h_\times$ waveforms for an inclination $i = 0°$ and a distance of 10 kpc, in units of 10^{-22}. The *bottom panels* display the Fourier transforms of the signals, in units of 10^{-23}

GH frekvencia – PN paraméter:

$$f_{GW} = \frac{Gm}{c^3} f_{GW} = \frac{2}{\tau_{orb}} = \frac{\varepsilon^{3/2}}{\pi}$$

$$m = 10^4 M_\odot$$

$$f_{GW} = 10^{-4}\ \text{Hz} \longrightarrow \varepsilon \approx 0.003$$

$$f_{GW} = 10^{-3}\ \text{Hz} \longrightarrow \varepsilon \approx 0.013$$

Időtartam a GH frekvencia változása között:

$$\Delta\tau = \frac{5}{256\eta}\left[\frac{1}{\left(\pi f_{GW}^{(in)}\right)^{8/3}} - \frac{1}{\left(\pi f_{GW}^{(fin)}\right)^{8/3}}\right]$$

$$f_{GW} = 10^{-4} \div 10^{-3}\ \text{Hz}$$
$$\varepsilon = 0.01 \longrightarrow \Delta\tau \approx 10^9$$

Flip-Flop periódus:

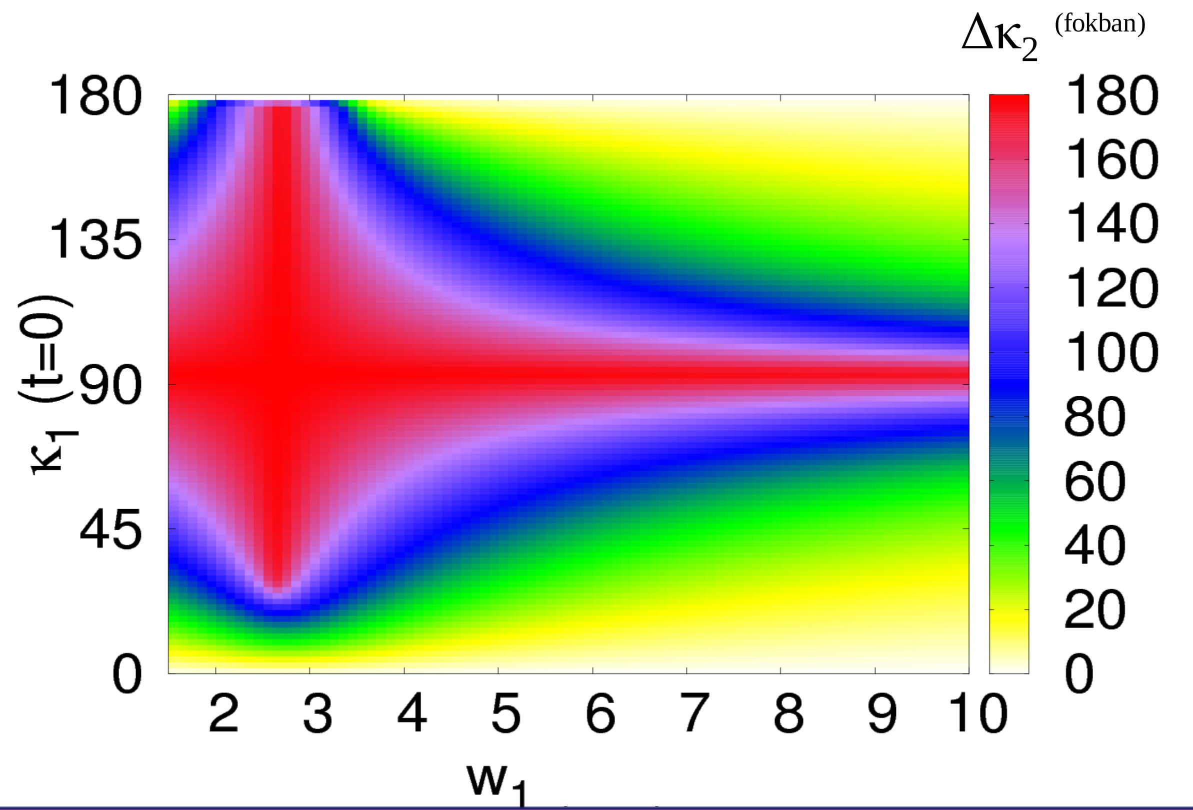
$$\varepsilon = 0.01 \qquad \mathfrak{T}_{ff} \approx 10^7$$

Kvadrupól paraméter hatása

WG Laarakers, E Poisson, Astrophys. J. **512**, 282 (1998)

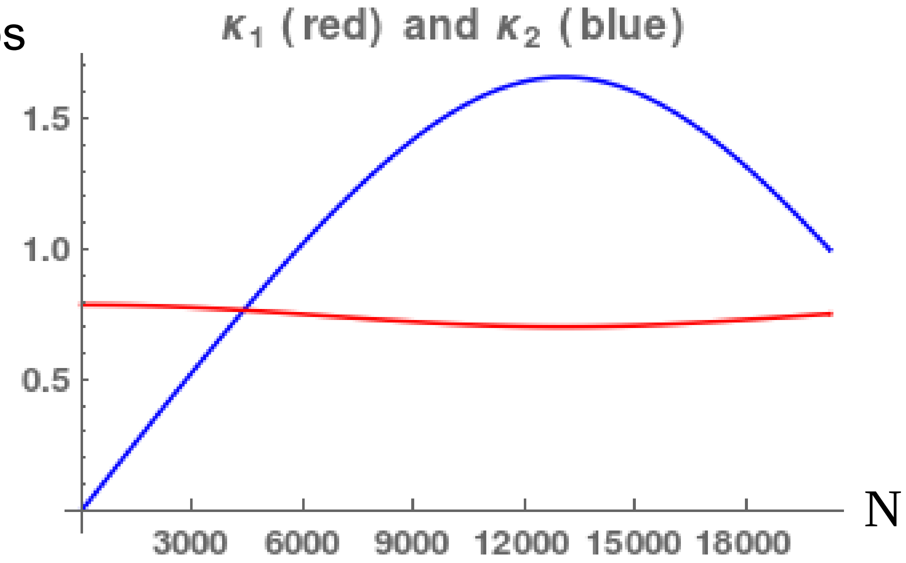
TABLE VII. Fit parameter

$M/M_\odot$	a =w(EOS,M)			
	EOS G	EOS FPS	EOS C	EOS L
1.0	5.1	7.8	9.4	12.1
1.2	3.3	5.7	6.9	9.3
1.4	2.0	4.3	5.0	7.4
1.6		3.1	3.7	6.0
1.8		2.2	2.7	4.9

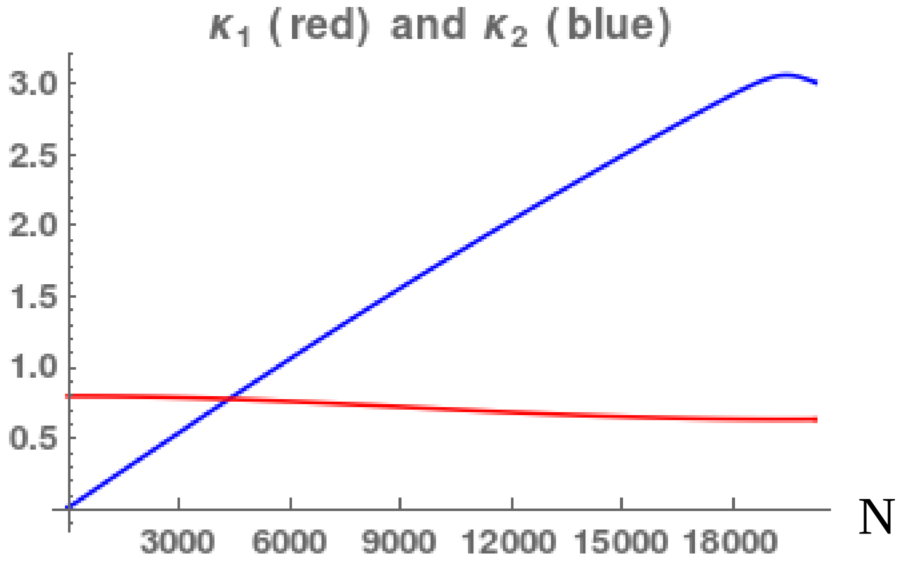


Kvadrupól paraméter hatása

Fekete lyuk kettős



W_1=2.7



Konklúziók

- Az excentricitás nincs hatással a flip-flop effektus nagyságára.
- Az excentricitás hatással van a flip-flop effektus periódusidejére.
- A kvadrupól paraméter jelentős hatással lehet a flip-flop effektus nagyságára.

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