Quantification of GR effects in muon g-2, EDM and other spin precession experiments

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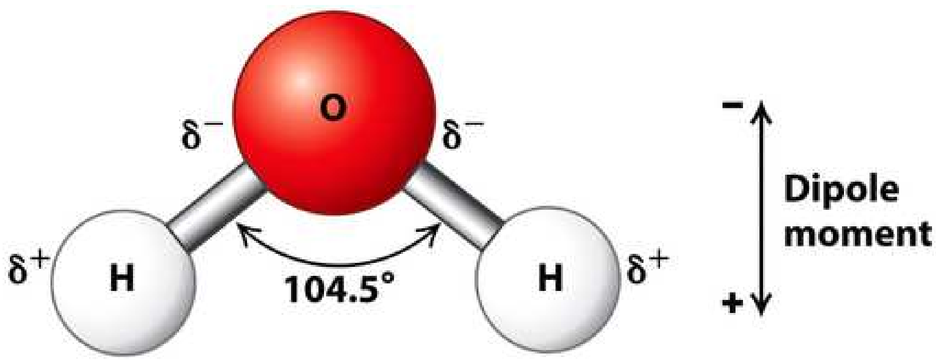
Introduction

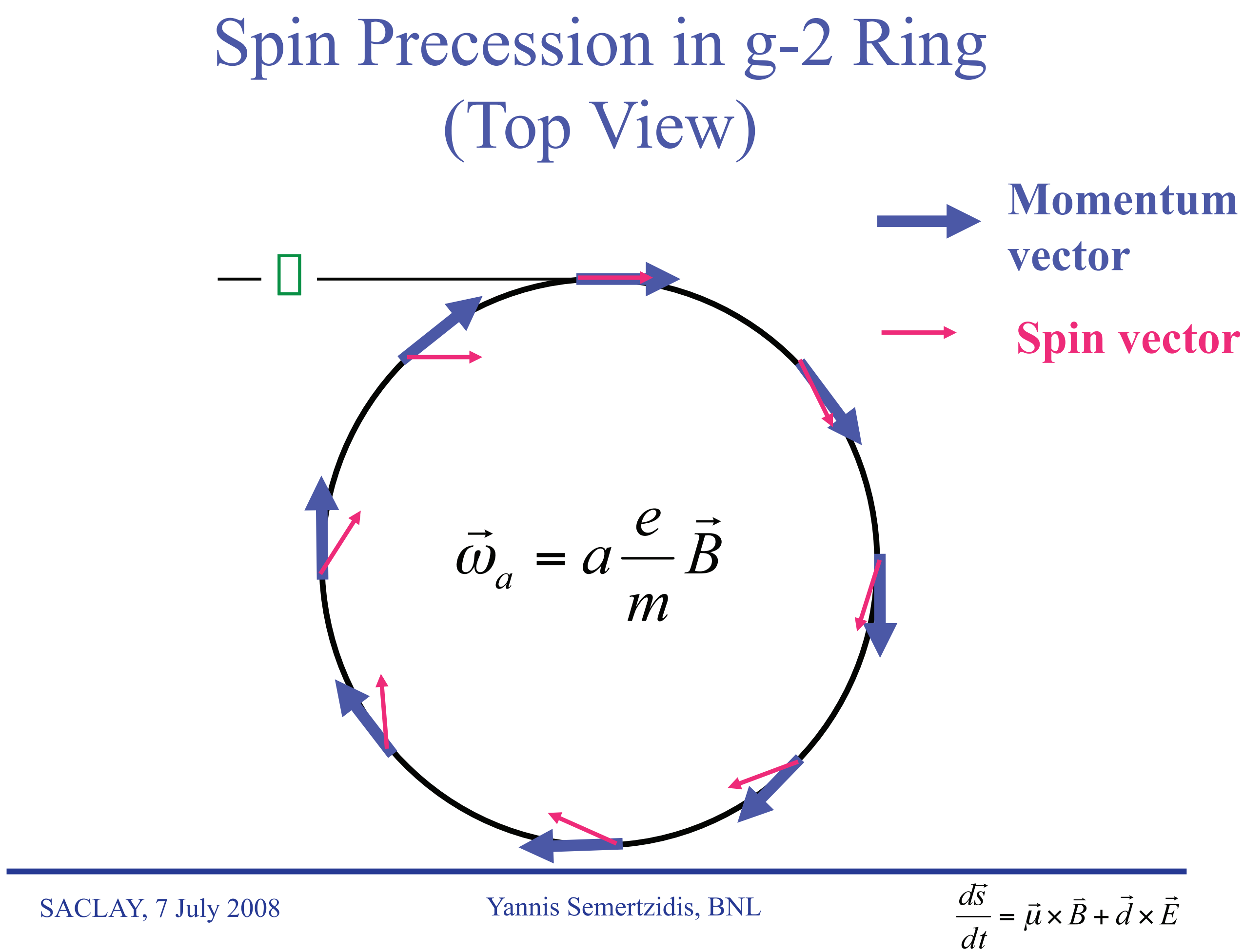
- Muon anomalous magnetic moment is known to be sensitive to radiative corrections.
  Classical Dirac equation: $\mathrm{g} := \frac{4\,m\,\mu}{q} = 2$.
  QFT: $\mathrm{g} \neq 2$ because of radiative corrections, sensitive to model content.
- Therefore, $\mathrm{g}-2$ is a sensitive probe for SM / BSM physics ($\mathrm{g}-2$ Collaboration).
  Experimental value: $a := \frac{(\mathrm{g}-2)}{2} \approx 0.0011659209\ldots$ is in $3\,\sigma$ disagreement with SM.

- Electric Dipole Moment (EDM) of particles are sensitive to their internal structures.
  Cartoon analogy: water molecule.



- Therefore, EDM is a sensitive probe for SM / BSM physics (CPEDM Collaboration).
  Planned experimental sensitivity: $10^{-29}\,\mathrm{V\,cm}$, physics reach $\approx 3000\,\mathrm{TeV}$ particle mass.

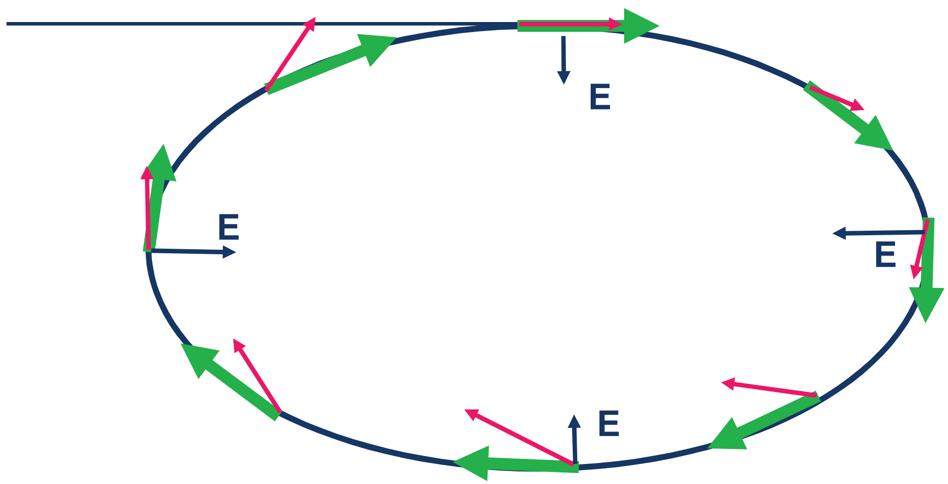
How muon $g-2$ is measured?



Particle goes around with circular frequency ω, spin lags with $a\,\omega$ from momentum.

How EDM is measured?

The spin precession relative to momentum in the plane is kept near zero. A vert. spin precession vs. time is an indication of an EDM (*d*) signal.



$$\vec{\omega}_a = 0 \qquad \frac{d\vec{s}}{dt} = \vec{d} \times \vec{E}$$

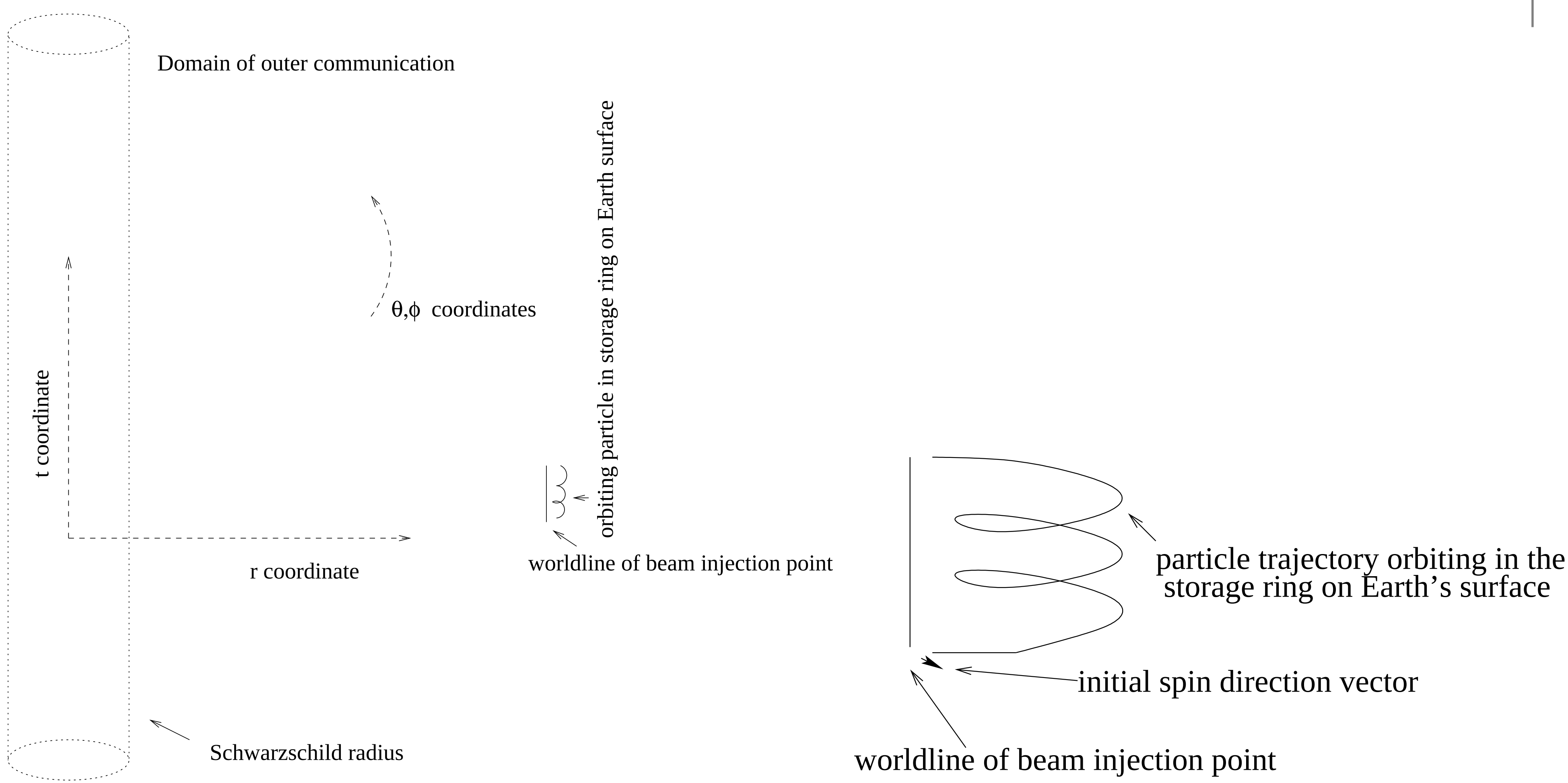
Yannis Semertzidis, BNL

Precession due to magnetic moment compensated by fields ("frozen spin"), rest is EDM.

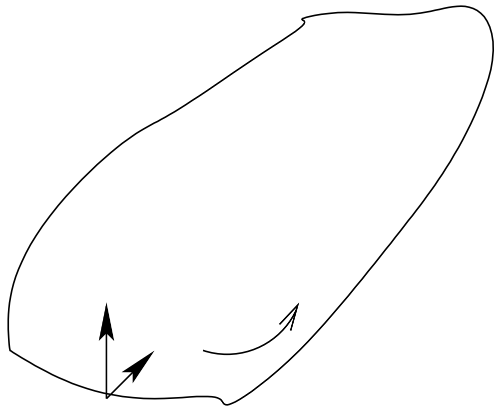
GR corrections:

- Recently a series of preprints appeared claiming that the General Relativity may give contribution to the muon $\mathrm{g}-2$ and generally to spin precession experiments.
  T. Morishima, T. Futamase, H. M. Shimizu:
  `arXiv:1801.10244`, `arXiv:1801.10245`, `arXiv:1801.10246`.
- Other authors claim that the effect is very small, much beyond the experimental resolution of 10^{-7} relative error.
  M. Visser: `arXiv:1802.00651`, P. Guzowski: `arXiv:1802.01120`.
- There are also other authors, who claim that the effect exactly cancels.
  H. Nikolic: `arXiv:1802.04025`.
- Which claims hold?
- It seems to be difficult to say something from first principles.
  (Also, formulas for the de Sitter and Lense-Thirring effect do not apply.)
- For EDM: there are some papers warning about possible GR effects.
  NPB911(2016)206, PRD76(2007)061101, PRD94(2016)044019, PLA376(2012)2822.
  Approximative calculations, qualitatively OK, quantitatively not OK against real GR.
- One info from first principles: only Earth's field matters.
  Because the system is free falling against other gravitating objects.

The kinematic setting



Due to transporting vectors on closed loops, the effect is very likely there.

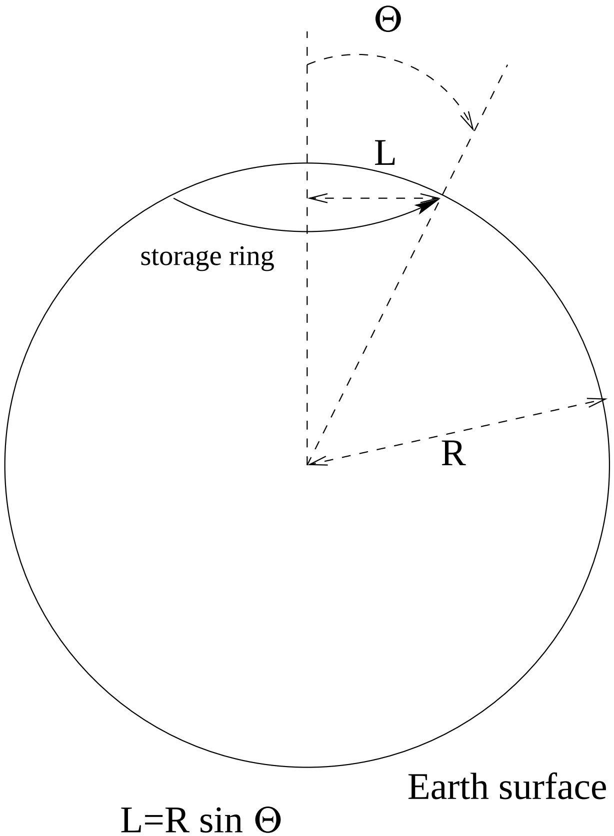


difference is expected due to curvature

Curvature is just measures this effect, but could be small.
("Wilson loop")

The only question seems to be: can such effect be large enough?

Coordinate conventions:



$$g_{\mathsf{ab}} = \begin{pmatrix} 1-\frac{r_S}{r} & 0 & 0 & 0 \\ 0 & -\frac{1}{1-\frac{r_S}{r}} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2\,\sin^2\vartheta \end{pmatrix}$$

Schwarzschild metric is time translationally (t) and spherically (ϑ, φ) symmetric.

Earth: $r = R = const$.

Storage ring: $r = R = const$, $\vartheta = \Theta = const$.

Elapse of time: measured by proper times. Curve parametrization: by Killing time t.

Worldline of orbiting muon:

$$\gamma_\omega{}^{\mathsf{a}}(t) \quad = \quad \begin{pmatrix} t \\ R \\ \Theta \\ \omega\sqrt{1-\frac{r_S}{R}}\, t \bmod 2\pi \end{pmatrix}$$

helical in spacetime.
Worldline of the beam injection point in the lab:

$$\gamma_0{}^{\mathsf{a}}(t) \quad = \quad \begin{pmatrix} t \\ R \\ \Theta \\ 0 \end{pmatrix}$$

a Killing time translation worldline.

The two intersect at full revolution times: $t = n\frac{2\pi}{\omega\sqrt{1-\frac{r_S}{R}}}$.

In these points, transported vectors could be compared.

The relativistic gyroscope equation

How spin direction vector evolves when forced on a worldline?
It is described by the *Fermi-Walker transport*:
Hawking-Ellis: *Large Scale Structure of Spacetime*; Cambridge University Press (1973).

It is a modified version of the parallel transport ∇, such that it conserves angles.

If u^a is a future directed unit timelike vector field, the F.-W. derivative of a vector field w^b is:

$$D_u^F w^b = u^d \nabla_d w^b + g_{ac} w^a u^b u^d \nabla_d u^c - g_{ac} w^a u^c u^d \nabla_d u^b$$

Properties:
(i) $D_u^F u^b = 0$, (ii) whenever $D_u^F w^b = 0$ and $D_u^F v^b = 0$ then $u^d \nabla_d (w_b v^b) = 0$ follows.

Relativistic gyroscope equation:

$$D_u^F w^b = 0$$

It is a parallel transport, taking into account the constraints

$$u_a u^b = 1, \quad w_a w^b = -1, \quad u_a w^b = 0.$$

See also: Gravity Probe B satellite experiment.

First we just solve the gyroscope equation for spin propagation along a forced circular orbit.

Then, we include the electromagnetic effects as well.
Full equation would be *Bargmann-Michel-Telegdi (BMT) equation*:

$$
\begin{aligned}
u^a \nabla_a u^b &= -\frac{q}{m} F^{ab} u_b && \longleftarrow \quad \text{(Newton eq.)} \\
D_u^F w^b &= -2\,\mu \left(F^{bc} - u^b u_d F^{dc} - F^{bd} u_d u^c \right) w_c && \longleftarrow \quad \text{(BMT eq.)}
\end{aligned}
$$

which causes *Larmor precession* in addition, due to F_{ab}.

This is used by physicists/engineers for beam optics design.

$$\frac{\mathrm{d}}{\mathrm{d}t} w_\omega{}^{\mathsf{b}}(t) \; + \; \dot\gamma_\omega{}^{\mathsf{d}}\, \Gamma^{\mathsf{b}}_{\mathsf{dc}}\, w_\omega{}^{\mathsf{c}}(t)$$

$$+ \, g_{\mathsf{ac}} w_\omega{}^{\mathsf{a}}(t) \frac{1}{\Lambda_\omega{}^2} \dot\gamma_\omega{}^{\mathsf{b}} \dot\gamma_\omega{}^{\mathsf{d}} \Gamma^{\mathsf{c}}_{\mathsf{de}} \dot\gamma_\omega{}^{\mathsf{e}} \; - \; g_{\mathsf{ac}} w_\omega{}^{\mathsf{a}}(t) \frac{1}{\Lambda_\omega{}^2} \dot\gamma_\omega{}^{\mathsf{c}} \dot\gamma_\omega{}^{\mathsf{d}} \Gamma^{\mathsf{b}}_{\mathsf{de}} \dot\gamma_\omega{}^{\mathsf{e}} \quad = \quad 0,$$

$$\frac{\mathrm{d}}{\mathrm{d}t} w_0{}^{\mathsf{b}}(t) \; + \; \dot\gamma_0{}^{\mathsf{d}}\, \Gamma^{\mathsf{b}}_{\mathsf{dc}}\, w_0{}^{\mathsf{c}}(t)$$

$$+ \, g_{\mathsf{ac}} w_0{}^{\mathsf{a}}(t) \frac{1}{\Lambda_0{}^2} \dot\gamma_0{}^{\mathsf{b}} \dot\gamma_0{}^{\mathsf{d}} \Gamma^{\mathsf{c}}_{\mathsf{de}} \dot\gamma_0{}^{\mathsf{e}} \; - \; g_{\mathsf{ac}} w_0{}^{\mathsf{a}}(t) \frac{1}{\Lambda_0{}^2} \dot\gamma_0{}^{\mathsf{c}} \dot\gamma_0{}^{\mathsf{d}} \Gamma^{\mathsf{b}}_{\mathsf{de}} \dot\gamma_0{}^{\mathsf{e}} \quad = \quad 0.$$

Because of the symmetries of the spacetime and of the orbit, this is a homogeneous linear differential equation with *constant coefficients*!

$$\Lambda_\omega \quad := \quad \sqrt{g_{\mathsf{ab}} \dot\gamma_\omega{}^{\mathsf{a}} \dot\gamma_\omega{}^{\mathsf{b}}},$$
$$\Lambda_0 \quad := \quad \sqrt{g_{\mathsf{ab}} \dot\gamma_0{}^{\mathsf{a}} \dot\gamma_0{}^{\mathsf{b}}}$$

in order to compensate Killing time $\leftrightarrow$ proper time.

Actually, the Fermi-Walker transport equation of any vector ${v_0}^b$ along the lab reference curve $t \mapsto \gamma_0(t)$ simplifies to $\frac{\mathrm{d}}{\mathrm{d}t}{v_0}^b(t) = 0$.

Thus, the second equation simplifies:

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}{w_\omega}^{\mathsf{b}}(t) \; + \; {\dot{\gamma}_\omega}^{\mathsf{d}}\, \Gamma^{\mathsf{b}}_{\mathsf{dc}}\, {w_\omega}^{\mathsf{c}}(t) & \\
+ \, g_{\mathsf{ac}} {w_\omega}^{\mathsf{a}}(t) \frac{1}{{\Lambda_\omega}^2} {\dot{\gamma}_\omega}^{\mathsf{b}} {\dot{\gamma}_\omega}^{\mathsf{d}} \Gamma^{\mathsf{c}}_{\mathsf{de}} {\dot{\gamma}_\omega}^{\mathsf{e}} \; - \; g_{\mathsf{ac}} {w_\omega}^{\mathsf{a}}(t) \frac{1}{{\Lambda_\omega}^2} {\dot{\gamma}_\omega}^{\mathsf{c}} {\dot{\gamma}_\omega}^{\mathsf{d}} \Gamma^{\mathsf{b}}_{\mathsf{de}} {\dot{\gamma}_\omega}^{\mathsf{e}} & = 0, \\
\frac{\mathrm{d}}{\mathrm{d}t}{w_0}^{\mathsf{b}}(t) & = 0.
\end{aligned}
$$

Homogeneous linear differential equations with constant coefficients!

Evolution equations:

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t} w_\omega{}^{\mathsf{b}}(t) &= \mathcal{F}_\omega{}^{\mathsf{b}}{}_{\mathsf{a}}\, w_\omega{}^{\mathsf{a}}(t), \\
\frac{\mathrm{d}}{\mathrm{d}t} w_0{}^{\mathsf{b}}(t) &= 0.
\end{aligned}
$$

Analytic expression of the evolution matrix:

$$
\mathcal{F}_\omega{}^{\mathsf{b}}{}_{\mathsf{c}} = \frac{\omega\sqrt{1-\frac{r_S}{R}}}{1-\omega^2 L^2}
$$

$$
\begin{pmatrix}
0 & -\omega L\frac{L}{R}\frac{\left(1-\frac{3}{2}\frac{r_S}{R}\right)}{\left(1-\frac{r_S}{R}\right)^{\frac{3}{2}}} & -\omega L R\frac{\sqrt{1-\left(\frac{L}{R}\right)^2}}{\left(1-\frac{r_S}{R}\right)^{\frac{1}{2}}} & 0 \\
-\omega L\frac{L}{R}\left(1-\frac{3}{2}\frac{r_S}{R}\right)\left(1-\frac{r_S}{R}\right)^{\frac{1}{2}} & 0 & 0 & L\frac{L}{R}\left(1-\frac{3}{2}\frac{r_S}{R}\right) \\
-\omega L\frac{1}{R}\sqrt{1-\left(\frac{L}{R}\right)^2}\left(1-\frac{r_S}{R}\right)^{\frac{1}{2}} & 0 & 0 & \frac{L}{R}\sqrt{1-\left(\frac{L}{R}\right)^2} \\
0 & -\frac{1}{R}\frac{\left(1-\frac{3}{2}\frac{r_S}{R}\right)}{\left(1-\frac{r_S}{R}\right)} & -\frac{R}{L}\sqrt{1-\left(\frac{L}{R}\right)^2} & 0
\end{pmatrix}
$$

$\mathcal{F}_\omega{}^{\mathsf{b}}{}_{\mathsf{c}} g^{\mathsf{ca}}$ is antisymmetric, thus $\mathcal{F}_\omega{}^{\mathsf{b}}{}_{\mathsf{a}}$ describes a Lorentz transformation generator. Also, $\mathcal{F}_\omega{}^{\mathsf{b}}{}_{\mathsf{a}}\ u_\omega{}^{\mathsf{a}} = 0$, thus it is an u_ω-rotation. (u_ω, u_0 are four velocities of γ_ω, γ_0.)

This is an absolute, observer independent effect, called **Thomas rotation**.

These provide possibilities for analytic crosscheck, which do pass.

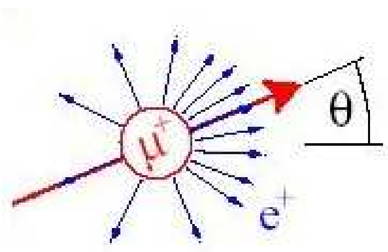
GRTensorII Maple package was used.

Precession as seen by observer

Actually, in the muon $\mathrm{g}-2$ experiment the spin orientation is measured by weak muon decay:

$$\mu \quad \longrightarrow \quad \nu_\mu + \bar{\nu}_e + e$$

Momentum conservation + angular momentum conservation + fixed helicity of neutrinos
$\Rightarrow$ the emission angle of e is correlated with muon spin (“self-polarimeter”).

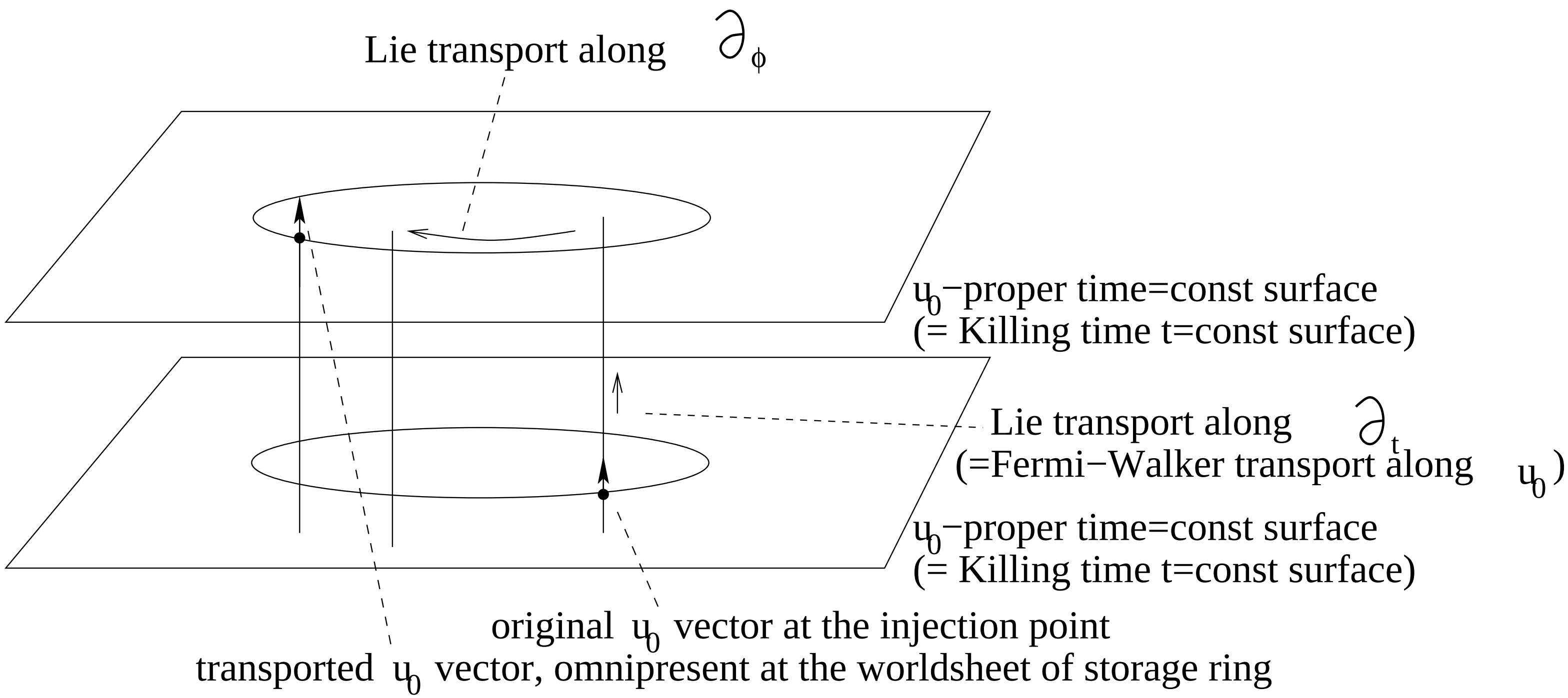


Incoming e signal is watched continuously as a function of timestamp to see precession.

Rate will vary according to $\sim \mathrm{e}^{-\frac{t}{\gamma\,\tau}}\,(1 + A\,\cos(t\,\omega_a + \phi))$ where ω_a is the revolution frequency of longitudinal projection of spin vector.

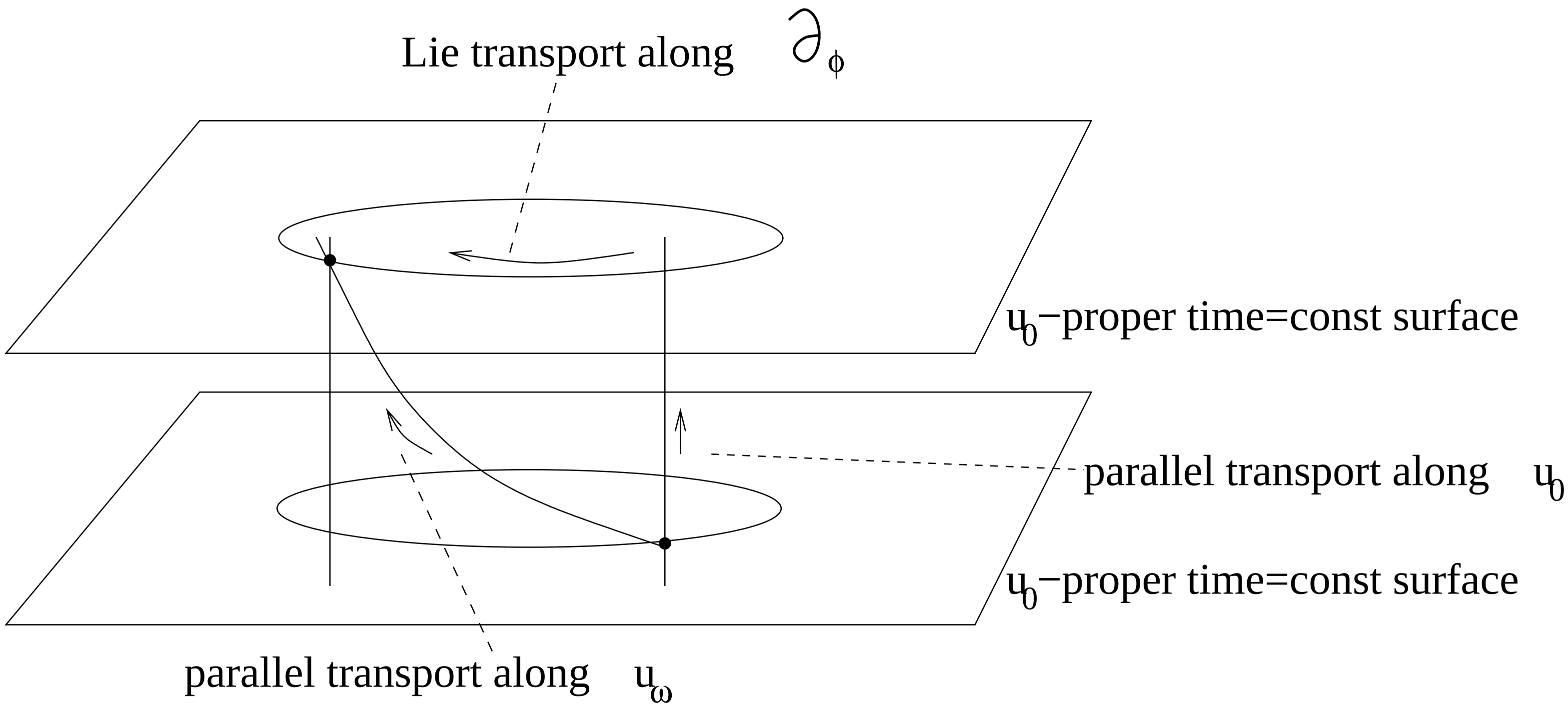
$\Rightarrow$ Einstein synchronized lab observer is necessary, not a local measurement.

Four velocity field of the injection point worldline u_0 can be spread along circumference:



"Lie extension": any vector can be spread in this way to the storage ring worldsheet
(v^a vector $\longrightarrow$ $\check{v}^a$ vector field, for which $\mathcal{L}_{\partial_t}\check{v}^a = 0$ and $\mathcal{L}_{\partial_\varphi}\check{v}^a = 0$ holds).

How the spread, Einstein synchronized u_0 observer measures time evolution?



By comparing u_ω parallel transport to u_0 parallel transport + ∂_φ Lie transport.

Time evolution of a vector field along curve γ_ω as seen by observer:

$$(v^a)' \;:=\; \frac{1}{u_{0\,b}\,u_\omega{}^b} u_\omega{}^c \nabla_c v^a - u_0{}^c \nabla_c \check{v}^a - \omega \mathcal{L}_{\partial_\varphi} \check{v}^a \;=\; \frac{1}{\Lambda_0}\frac{\mathrm{d}}{\mathrm{d}t} v^a + \frac{1}{\Lambda_0}\left(\dot{\gamma}_\omega^b - \dot{\gamma}_0^b\right)\Gamma^a_{bc} v^c.$$

How the spread, Einstein synchronized u_0 observer views moving three-vectors?

$$\begin{array}{ccccc} {w_\omega}^a & := & {B_{u_\omega,u_0}}^a{}_b & {w_{\omega,u_0}}^b \\ \uparrow & & \uparrow & \uparrow \\ \text{absolute} & & \text{boost} & \text{as seen by observer} \\ \text{(obeys gyroscope eq.)} & & & \end{array}$$

${B_{u_\omega,u_0}}^a{}_b := \delta^a{}_b - \frac{({u_\omega}^a+{u_0}^a)(u_{\omega\, b}+u_{0\, b})}{1+u_{\omega\, c}\, {u_0}^c} + 2\, {u_\omega}^a\, u_{0\, b}$ is the Lorentz boost from u_0 to u_ω.

Putting all this together, the relative gyroscope equation is:

$$\left({w_{\omega,u_0}}^{\mathsf{f}}\right)' = {\Phi^T_{\omega,u_0}}^{\mathsf{f}}{}_{\mathsf{b}} \quad {w_{\omega,u_0}}^{\mathsf{b}}$$

with

$${\Phi^T_{\omega,u_0}}^{\mathsf{f}}{}_{\mathsf{b}} = \frac{1}{\Lambda_0}\left(\dot{\gamma}^{\mathsf{d}}_\omega - \dot{\gamma}^{\mathsf{d}}_0\right)\Gamma^{\mathsf{f}}_{\mathsf{db}} + \frac{1}{\Lambda_0}\, {B_{u_0,u_\omega}}^{\mathsf{f}}{}_{\mathsf{a}}\, {\mathcal{F}_\omega}^{\mathsf{a}}{}_{\mathsf{e}}\, {B_{u_\omega,u_0}}^{\mathsf{e}}{}_{\mathsf{b}}$$

Analytically seen: $\Phi^{T}_{\omega,u_0}{}^{\mathsf{f}}{}_{\mathsf{b}}$ is an u_0-rotation generator.

It describes a relative phenomenon, called **Thomas precession**.

Best presented via its u_0 Hodge dual (angular velocity vector):

$$\Omega^{T}_{\omega,u_0}{}^{\mathsf{f}} \quad := \quad \frac{1}{2}\, u_0{}^{\mathsf{a}} \sqrt{-\det(g)}\, \epsilon_{\mathsf{abcd}}\, g^{\mathsf{bf}}\, \Phi^{T}_{\omega,u_0}{}^{\mathsf{c}}{}_{\mathsf{e}}\, g^{\mathsf{ed}}.$$

If d^a is a corotating vector field along γ_ω, i.e. $\frac{\mathrm{d}}{\mathrm{d}t} d^a = 0$, then:

$$\left(d^{\mathsf{f}}\right)' \quad = \quad \Phi^{C}_{\omega,u_0}{}^{\mathsf{f}}{}_{\mathsf{b}} \quad d^{\mathsf{b}}$$

with

$$\Phi^{C}_{\omega,u_0}{}^{\mathsf{f}}{}_{\mathsf{b}} \quad = \quad \frac{1}{\Lambda_0} \left(\dot\gamma_\omega^{\mathsf{d}} - \dot\gamma_0^{\mathsf{d}}\right) \Gamma^{\mathsf{f}}_{\mathsf{db}}$$

because of the definition of $(\cdot)'$. It has corresponding $\Omega^{C}_{\omega,u_0}{}^{\mathsf{f}}$. **Cyclic motion**.

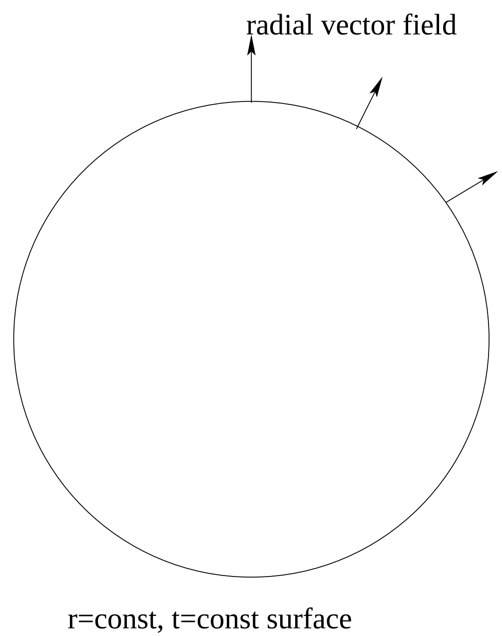
E.g.: $d^a = \hat\varphi^a$, then $\hat\varphi_a\, w_{\omega,u_0}{}^a$ will oscillate with $\left|\Omega^{T}_{\omega,u_0} - \Omega^{C}_{\omega,u_0}\right|$. $\longrightarrow$ observable of $\mathrm{g}-2$!

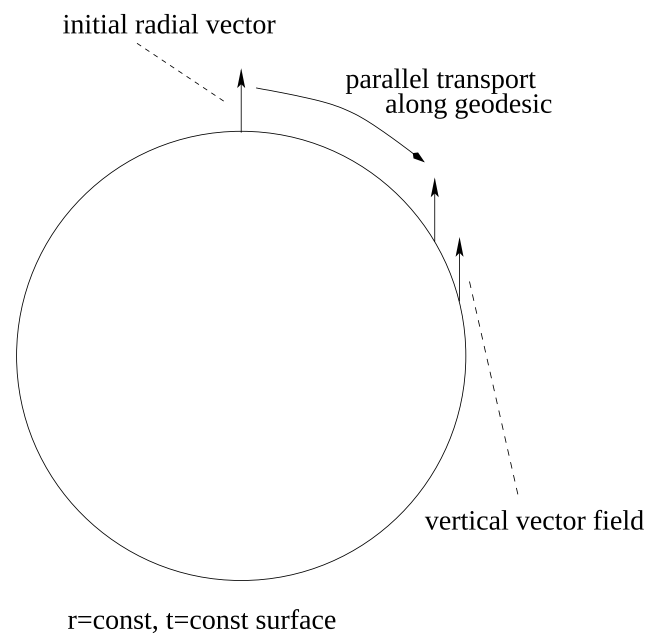
The electromagnetic (Larmor) precession

The particles orbit in a vertical magnetic field of storage ring.

Let $\hat{r}^a$ be the outward pointing unit radial vector field of $r = const$ surface.

From this one can construct the vertical vector field $\hat{v}^a$:





(Parallel transport in terms of induced metric $g_{ab} + \hat{r}_a \hat{r}_b - \mathrm{d}r_a \mathrm{d}r_b$.)

Homogeneous magnetic field in Schwarzschild: ansatz with

$$B^a \quad := \quad b(r)\,\hat{v}^a$$

where F^B_{ab} is the u_0 Hodge dual of B^a, and we require it to solve vacuum Maxwell.

Beam optics has electrostatic quadrupole focusing
$\Rightarrow$ on average a radial electric field balancing the beam against falling towards the Earth.

$$E^a \quad := \quad e(r)\,\hat{r}^a$$

where $F^E_{ab} := u_{0\,a}\,E_b - u_{0\,b}\,E_a$, and we require it to solve vacuum Maxwell.

$$B^a \quad = \quad B\,\sqrt{\frac{1-\frac{r_S}{r}}{1-\frac{r_S}{R}\,\frac{L^2}{R^2}}}\,\hat{v}^a \qquad \leftarrow \text{homogeneous vertical}$$

$$E^a \quad = \quad E\,\frac{R^2}{r^2}\,\hat{r}^a \qquad \leftarrow \text{radial Coulomb}$$

Total electromagnetic field: $F_{ab} := F^B_{ab} + F^E_{ab}$.

From Newton equation ${u_\omega}^a \nabla_a {u_\omega}^b = -\frac{q}{m}\, F^{bc}\, u_{\omega\, c}$:

$$\begin{aligned} B &= \omega\, \frac{m\,\gamma}{q}\, \sqrt{1 - \frac{r_S}{R}\, \frac{L^2}{R^2}}, \\ E &= \frac{r_S}{2\,R^2}\, \frac{m\,\gamma}{q}\, \frac{1}{\sqrt{1 - \frac{r_S}{R}}} \end{aligned}$$

with $\gamma := \frac{1}{\sqrt{1-\beta^2}}$, $\beta := \omega\, L$. **Cyclotronic motion** over Schwarzschild.

From BMT equations $D^F_u w^b = -2\,\mu\, \left(F^{bc} - u^b u_d F^{dc} - F^{bd} u_d u^c\right)\, w_c$:

$$\frac{\mathrm{d}}{\mathrm{d}t} {w_\omega}^b = \left({\mathcal{F}_\omega}^b{}_c + {L_\omega}^b{}_c\right) {w_\omega}^c$$

with ${L_\omega}^b{}_c := -\Lambda_\omega\, 2\,\mu \left(F^{bc} - u^b u_d F^{dc} - F^{bd} u_d u^c\right)$.

Direct calculation: ${\mathcal{F}_\omega}^b{}_c + {L_\omega}^b{}_c = -a\, {\mathcal{F}_\omega}^b{}_c$,

with $\mathrm{g} := \frac{4\,m\,\mu}{q}$ and $a := \frac{\mathrm{g}-2}{2}$.

Thomas + Larmor precession as seen by observer

$$\left(w_{\omega,u_0}{}^{\mathsf{f}}\right)' = \left(\Phi^T_{\omega,u_0}{}^{\mathsf{f}}{}_{\mathsf{b}} + \Phi^L_{\omega,u_0}{}^{\mathsf{f}}{}_{\mathsf{b}}\right) \quad w_{\omega,u_0}{}^{\mathsf{b}}$$

with

$$\Phi^L_{\omega,u_0}{}^{\mathsf{f}}{}_{\mathsf{b}} := \frac{1}{\Lambda_0} B_{u_0,u_\omega}{}^f{}_c \, L_\omega{}^c{}_a \, B_{u_\omega,u_0}{}^a{}_b .$$

Total effect:

$$\begin{aligned}
\hat{t}_a \, \Omega^S_{\omega,u_0}{}^a &= 0 \\
-\hat{r}_a \, \Omega^S_{\omega,u_0}{}^a &= -\omega \, (1 + a\,\gamma) \sqrt{1 - \frac{L^2}{R^2}} \\
-\hat{\vartheta}_a \, \Omega^S_{\omega,u_0}{}^a &= \omega \, \frac{L}{R} \left(\sqrt{1 - \frac{r_S}{R}} + a\,\gamma \, \frac{1 - \frac{3}{2}\frac{r_S}{R}}{\sqrt{1 - \frac{r_S}{R}}} \right) \qquad \leftarrow !!! \\
-\hat{\varphi}_a \, \Omega^S_{\omega,u_0}{}^a &= 0.
\end{aligned}$$

Total effect on oscillation frequency of $(\hat{\varphi}_a \, w_{\omega,u_0}{}^a)$:

$$\left| \Omega^S_{\omega,u_0} - \Omega^C_{\omega,u_0} \right| = |\omega\, a| \, \gamma \sqrt{1 - \frac{L^2}{R^2} + \frac{L^2}{R^2} \, \frac{\left(1 - \frac{3}{2}\frac{r_S}{R}\right)^2}{1 - \frac{r_S}{R}}} \qquad \leftarrow !!!$$

Evaluation

From first order Taylor term of $\left|\Omega^S_{\omega,u_0} - \Omega^C_{\omega,u_0}\right|$ in terms of r_S, in ultrarelativistic limit:

$$-\frac{1}{2}\,\frac{r_S}{R}\,\frac{L^2}{R^2} \;\approx\; -10^{-21}$$

is the relative error, negligible for muon $\mathrm{g}{-}2$, or any $\mathrm{g}{-}2$.

But! For EDM, it is not negligible.
For “frozen spin” situation $\;1 + a\,\gamma = 0\;$ one has ${\Omega^S_{\omega,u_0}}^a = 0$ for Minkowski limit, however:

$$(-\hat{\vartheta}_a\, {\Omega^S_{\omega,u_0}}^a) \quad\approx\quad \beta\,\frac{r_S}{2\,R^2} \;\approx\; 3.3\cdot 10^{-8}\,\mathrm{rad/sec}$$

which is above planned EDM sensitivity limit.

Remark:
For deuterons, $a \approx -0.14$, and thus $\;1 + a\,\gamma = 0\;$ possible, i.e. $\left.{\Omega^S_{\omega,u_0}}^a\right|_{r_S=0} = 0$ possible.
Experimentally, they prefer to freeze longitudinal spin.
Possible for any a, with a combination of γ + electric field + magnetic field.

Conclusion

- GR effects are negligible for $\mathrm{g}-2$ experiments.
- GR effects are not negligible for EDM or other “frozen spin” experiments.

  Planned EDM experiments with sensitivity $\approx 10^{-29}\,\mathrm{V\,cm}$, i.e. of 1 nrad/sec can serve as GR experiments.

  Like microscopic Gravity Probe B experiments.