Measuring topological invariants using losses

Tibor Rakovszky[1], Janos Asboth[2], Andrea Alberti[3]

1: TU München
2: Wigner Research Centre for Physics, Budapest
3: Universitaet Bonn

[Phys. Rev. B 95, 201407 (2017)]

University of Szeged, Theroetical Physics Seminar, 2019 February 19

Plan for the next hour

- Intro 1: - Chiral symmetry
  - Topologically protected edge state
  - Bulk topological invariants

  [Asboth, Palyi, Oroszlany, Short Course on Topological Insulators]

- Intro 2: - Nonhermitian Hamiltonian
  - Decay position as an observable

  [Rudner & Levitov, PRL (2009)]

- Our work: - Generalizing to periodically driven systems
  - Exact results for disorder

  [Rakovszky, Asboth, Alberti, PRB (2017)]

- Open questions

- Intro 1: - Chiral symmetry
  - Topologically protected edge state
  - Bulk topological invariants

  [Asboth, Palyi, Oroszlany, Short Course on Topological Insulators]

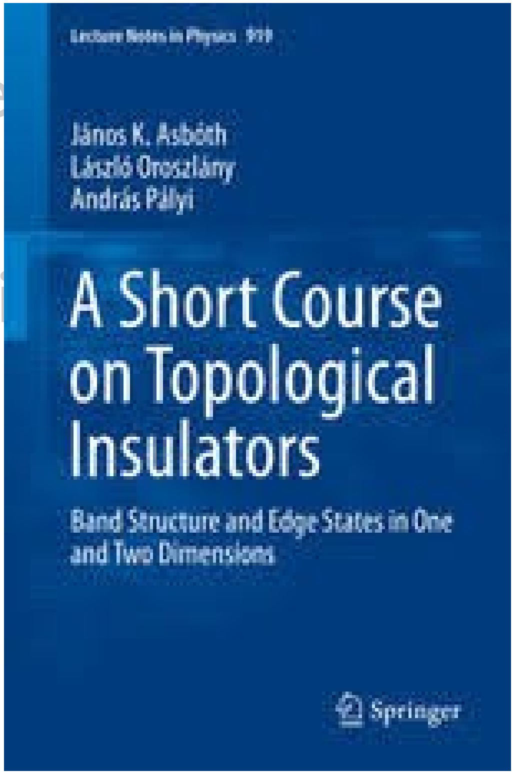
- Intro 2: - Nonhermitian Hamiltonian
  - Decay position as an observable

  [Rudner & Levitov, PRL (2009)]

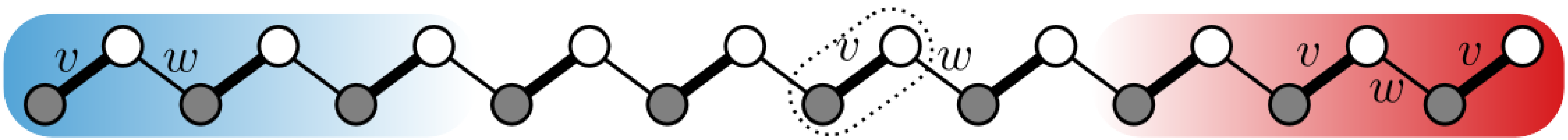
- Our work: - Generalizing to periodically dri
  - Exact results for disorder

  [Rakovszky, Asboth, Alberti, PRB (2017)]

- Open questions



Simplest example for topological insulator: Su-Schrieffer-Heeger model of polyacetylene



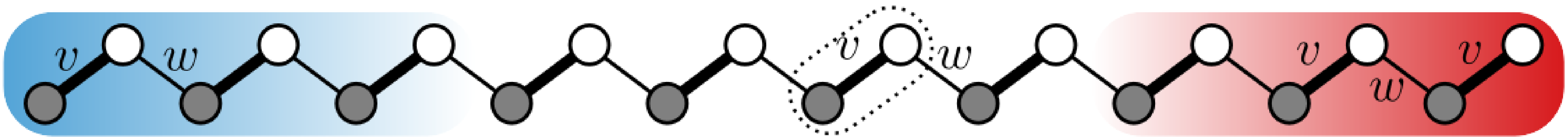
$$\hat{H} = v \sum_{m=1}^{L} (|m,B\rangle\langle m,A| + h.c.) + w \sum_{m=1}^{L-1} |m,B\rangle\langle m+1,A| + h.c.)$$

Nearest neighbor hopping, no onsite energies

The simpler cousin of Kitaev wire

Majorana zero modes	→	Zero-energy edge states
protected, superconductivity	→	protected, chiral symmetry

Sublattice symmetry = chiral symmetry of the Su-Schrieffer-Heeger model



$$\hat{H} = v \sum_{m=1}^{L} \left(|m, B\rangle\langle m, A| + h.c. \right) + w \sum_{m=1}^{L-1} |m, B\rangle\langle m+1, A| + h.c.)$$

Define sublattice projectors A, B, symmetry operator Γ

$$\hat{\Pi}_A = \frac{1 + \hat{\Gamma}}{2}$$

$$\hat{\Gamma} = \hat{\Pi}_A - \hat{\Pi}_B = \sum_{m=1}^{N} \left(|m, A\rangle\langle m, A| - |m, B\rangle\langle m, B| \right)$$

$$\hat{\Pi}_B = \frac{1 - \hat{\Gamma}}{2}$$

No transitions between sites on the same sublattice:

$$\hat{\Gamma}\hat{H}\hat{\Gamma} = -\hat{H} \qquad \Gamma = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix} \qquad H(k) = \begin{pmatrix} 0 & h(k) \\ h^\dagger(k) & 0 \end{pmatrix}$$

Chiral symmetry:
Eigenstates with E≠-E equal weight on A, B sublattices
Eigenstates with E=-E confined to one sublattice

$$\hat{H}|\Psi_n\rangle = E_n|\Psi_n\rangle$$

$$\hat{H}\hat{\Gamma}|\Psi_n\rangle = -\hat{\Gamma}\hat{H}|\Psi_n\rangle = -E_n|\Psi_n\rangle$$

Symmetric spectrum:

$$E_{-n} = -E_n$$

Γ gives chiral partner:

$$\Gamma|\Psi_n\rangle = e^{i\phi_n}|\Psi_{-n}\rangle$$

$$E_n \neq -E_n \Longrightarrow \quad \langle\Psi_n|\hat{\Gamma}|\Psi_n\rangle = 0 \Longrightarrow \quad \langle\Psi_n|\hat{\Pi}_A|\Psi_n\rangle = \langle\Psi_n|\hat{\Pi}_B|\Psi_n\rangle = \frac{1}{2}$$

$$E_n = -E_n \Longrightarrow \quad \hat{\Pi}_{A/B}|\Psi_n\rangle \propto |\Psi_n\rangle \pm \hat{\Gamma}|\Psi_n\rangle$$

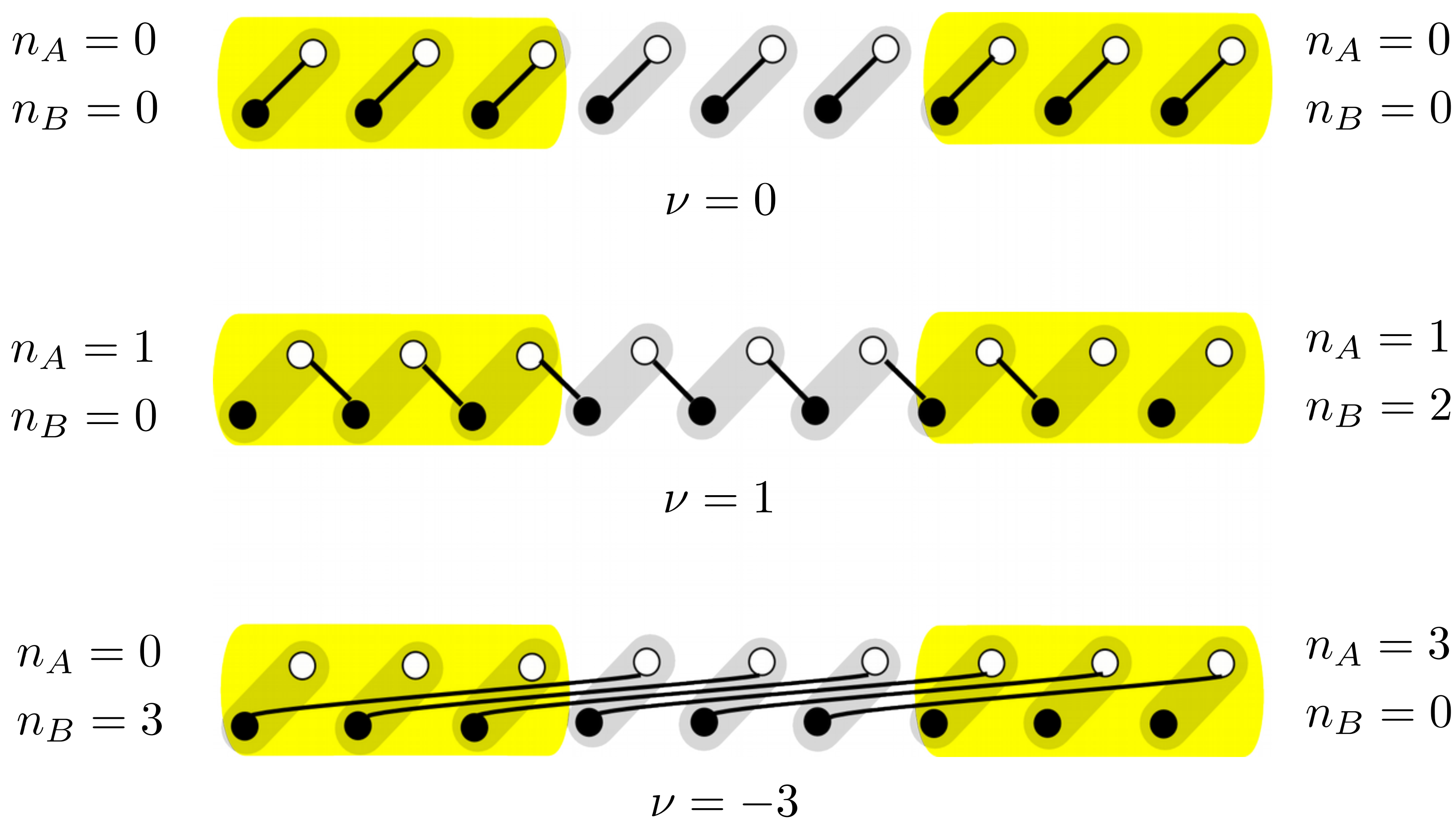
Energy eigenstate on a single sublattice

Bulk sublattice polarization predicts number of end states

Left end: unpaired sites

Bulk: sublattice A shifted by ν unit cells

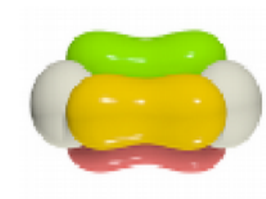
Right end: unpaired sites



Bulk sublattice polarization = winding number ν

Bulk polarization identified with Zak phase:

$$P = \frac{1}{2\pi i} \sum_{n<0} \int_{BZ} dk \langle n(k) | \frac{d}{dk} | n(k) \rangle$$



Projected to a single sublattice:

$$P_A = \frac{1}{2\pi i} \sum_{n<0} \int_{BZ} dk \langle n(k) | \hat{\Pi}_A \frac{d}{dk} \hat{\Pi}_A | n(k) \rangle$$

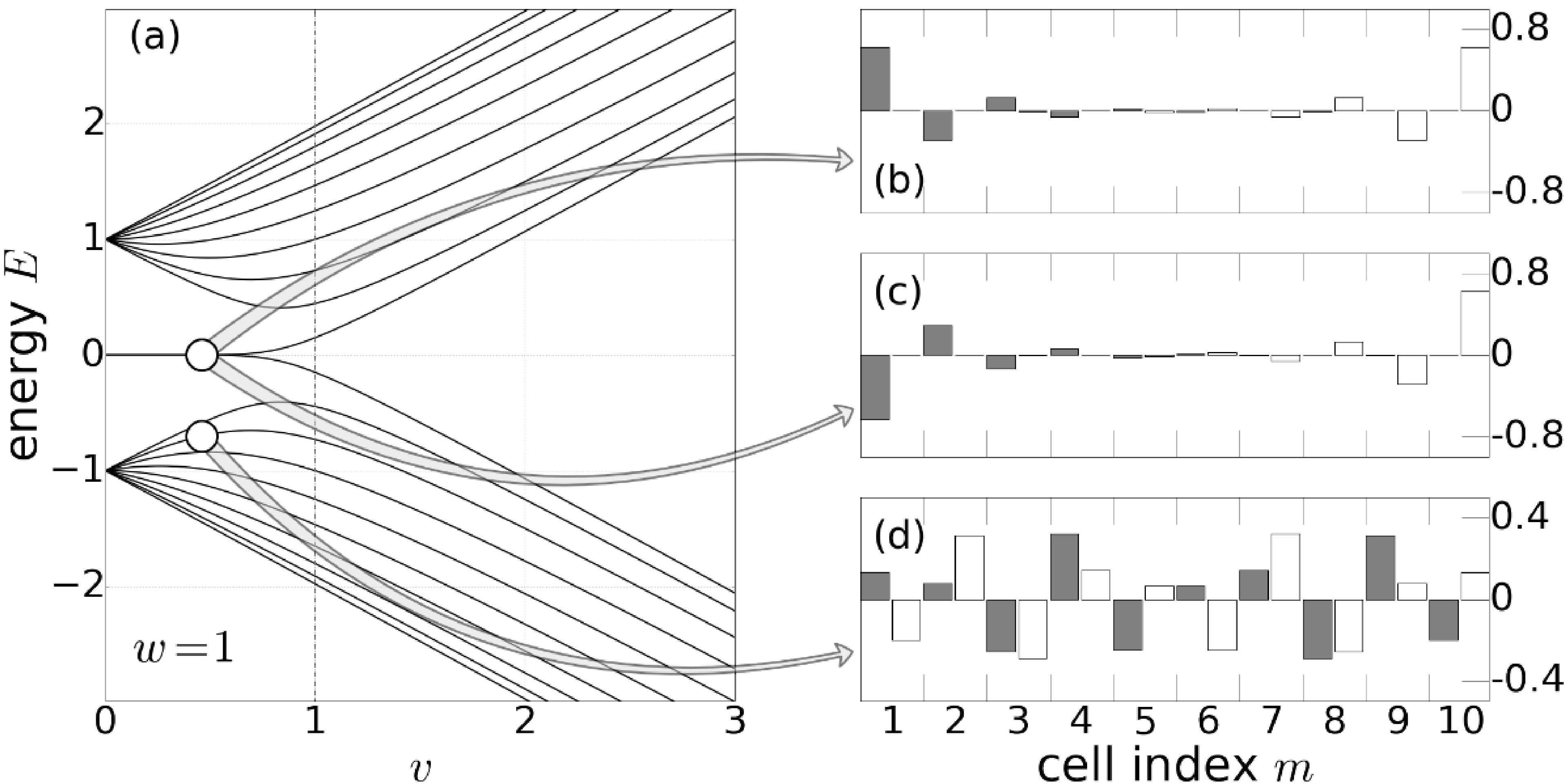
Sublattice polarization:

$$P_A - P_B = \frac{1}{2\pi i} \int_{BZ} dk \frac{d}{dk} \log \det h(k) \equiv \nu[h]$$

$$\Gamma = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix} \qquad H(k) = \begin{pmatrix} 0 & h(k) \\ h^\dagger(k) & 0 \end{pmatrix}$$

Details: Mondragon-Shem et al, PRL 113, 046802 (2014)

Edge states on one sublattice pinned to 0 energy by chiral symmetry



- Intro 1: - Chiral symmetry
  - Topologically protected edge state
  - Bulk topological invariants

  [Asboth, Palyi, Oroszlany, Short Course on Topological Insulators]

- Intro 2: - Nonhermitian Hamiltonian
  - Decay position as an observable

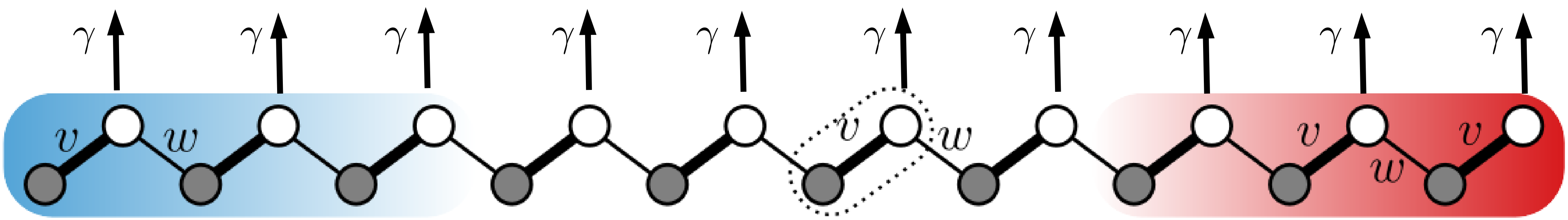
  [Rudner & Levitov, PRL (2009)]

- Our work: - Generalizing to periodically driven systems
  - Exact results for disorder

  [Rakovszky, Asboth, Alberti, PRB (2017)]

- Open questions

Rudner and Levitov (2009): Nonhermitian SSH, sublattice B has decay channels

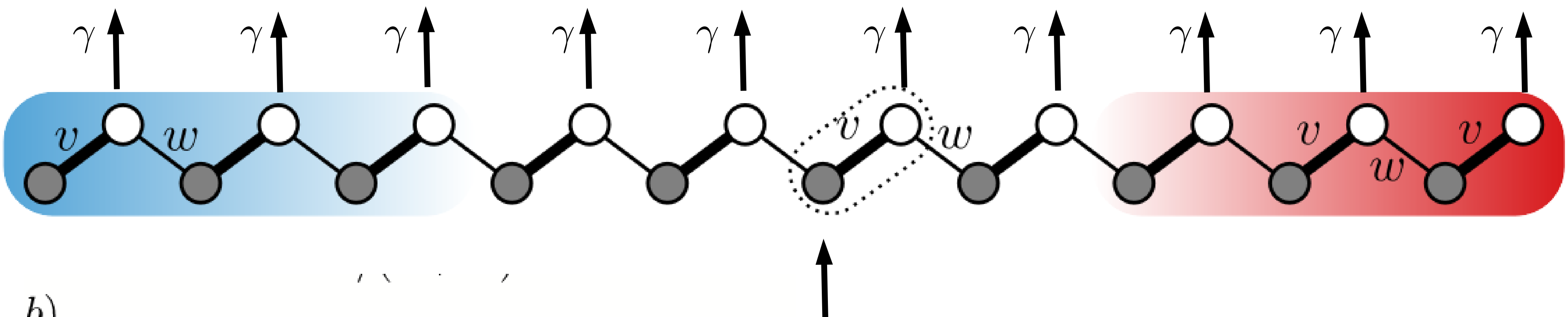


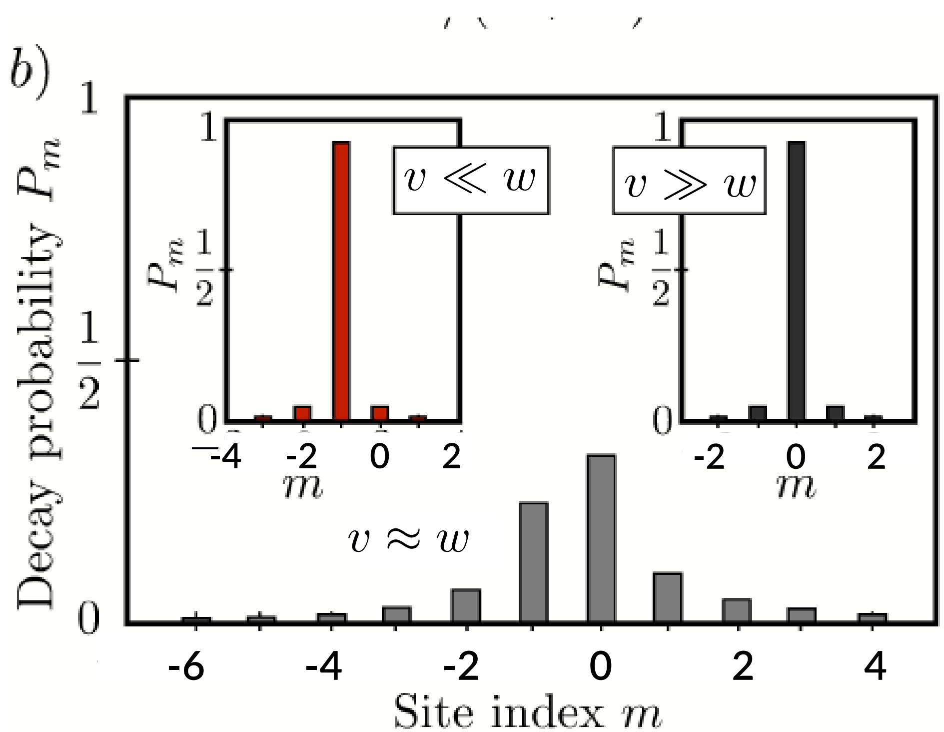
$$\hat{H} = v \sum_{m=1}^{L} (|m, B\rangle\langle m, A| + h.c.) + w \sum_{m=1}^{L-1} |m, B\rangle\langle m+1, A| + h.c.) - i\gamma \sum_{m=1}^{L} |m, B\rangle\langle m, B|$$

Nonhermitian Hamiltonian for conditional time evolution.
Condition: no decay events.
Norm of wavefunction = prob(condition holds)

Rudner and Levitov (2009): Nonhermitian SSH, expected displacement until decay = top. inv.

When decay happens, collect particle. Position of decay=displacement until decay





Insert single particle
at m=0, A

$$\bar{m} = \langle \Delta x \rangle = \nu$$

topological proof: mapping to a winding number

Our questions

- Is Rudner & Levitov result general, or only specific to two-band model? (Their proof only works for two-band model)
- Is it valid for disordered systems?
- How to translate this to periodically driven systems?

$$\hat{H}(t) = \hat{H}(t+1) \qquad \hat{U} = \mathcal{T} e^{-i\int_0^1 \hat{H}(t)dt} = e^{-i\hat{H}_{\text{eff}}}$$

energy → quasienergy E

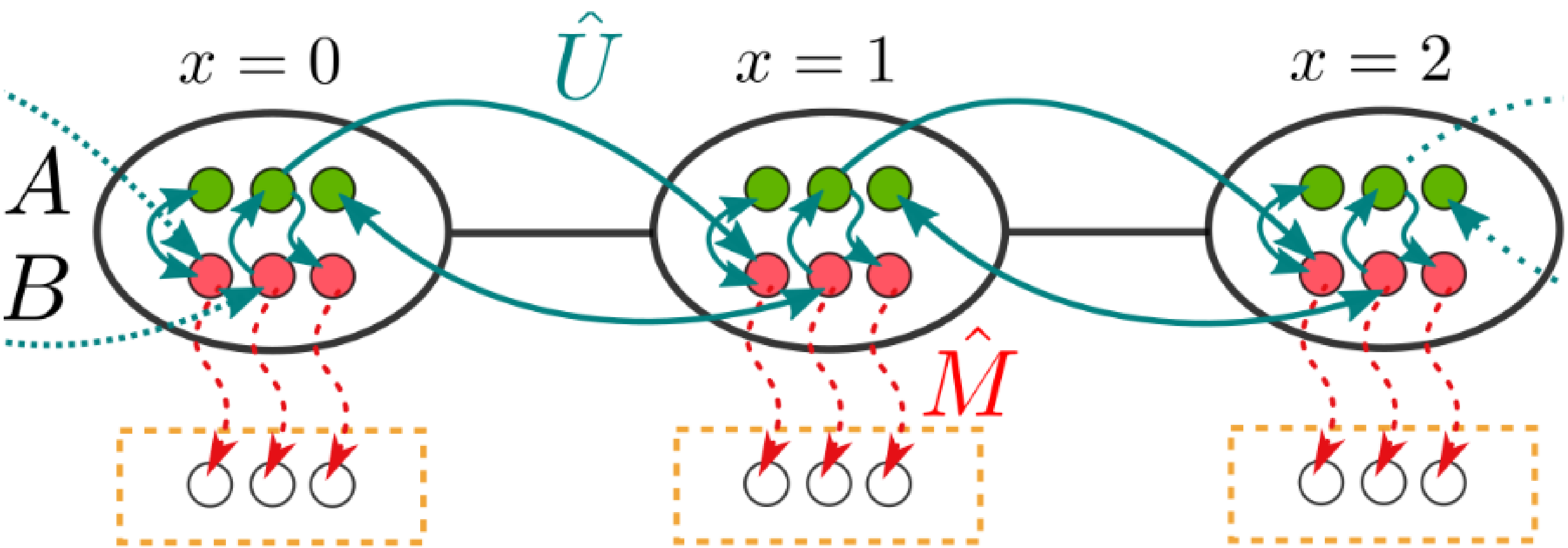
chiral symmetry → unitary time reversal $\quad \hat{\Gamma}\hat{U}\hat{\Gamma} = \hat{U}^\dagger$

pair of winding numbers at E=0, E=π [Asboth & Obuse, PRB (2013)]

1) Do everything for periodically driven systems

2) Recover non-Hermitian Hamiltonians as limiting case

Weak measurement on sublattice B at the end of each driving cycle



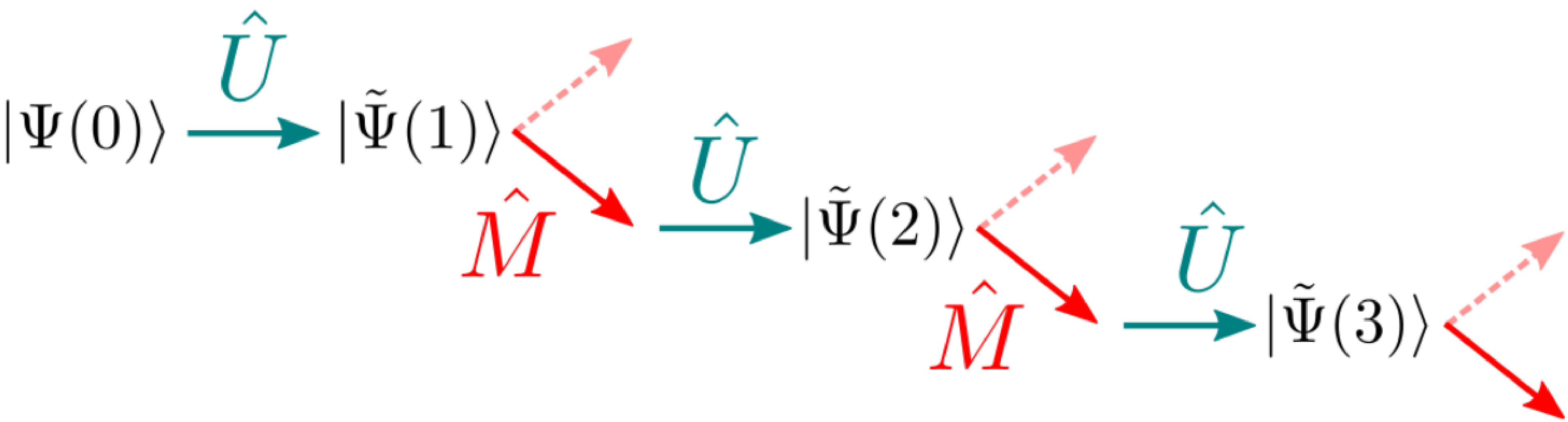
Effect of negative measurement:

(particle <u>not</u> detected)

$$\hat{M} = \hat{P}_A + \sqrt{1 - p_M}\,\hat{P}_B$$

Measurement efficiency

Continue time evolution until particle is detected



Conditional wavefunction: $$|\tilde{\Psi}(t)\rangle = \hat{U}\left[\hat{M}\hat{U}\right]^{t-1}|\Psi(0)\rangle$$

$$\hat{M} = \hat{P}_A + \sqrt{1-p_M}\,\hat{P}_B$$

Static case: $\text{period time} \to 0,\ p_M \to 0$

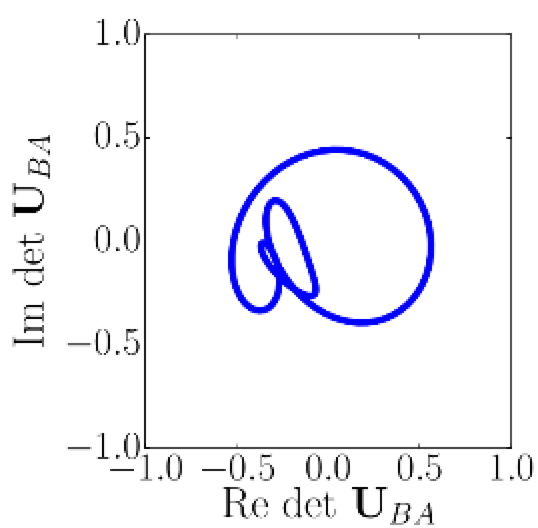
Expected displacement ⟨Δx⟩= υ

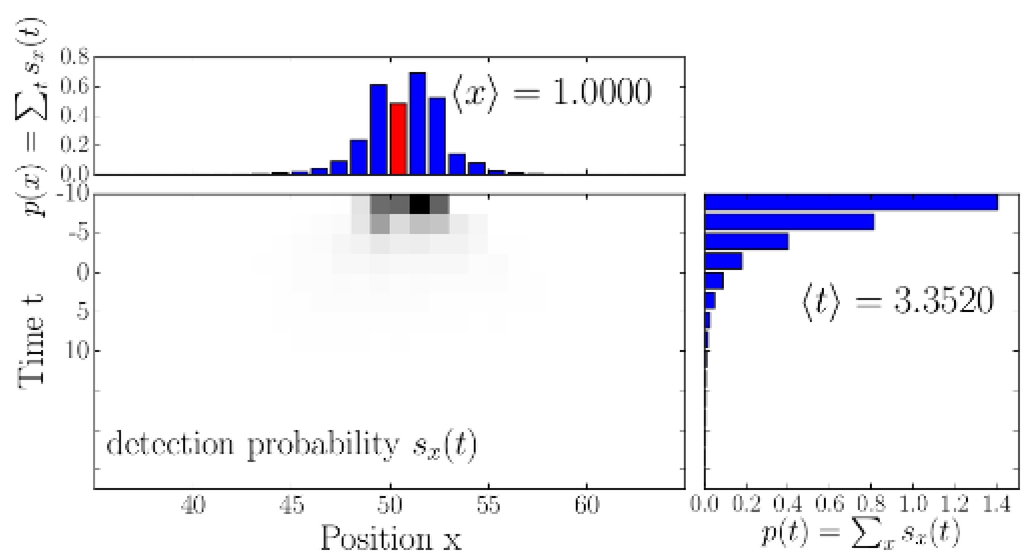
Expectation value of measured position:

$$\langle x \rangle \equiv \frac{p_M}{N} \sum_{t\in\mathbb{Z}^+} \sum_{x\in\mathbb{Z}} x \sum_{b=N+1}^{2N} \sum_{a=1}^{N} \left| \langle x, b | \hat{U} [\hat{M}\hat{U}]^{t-1} | x_0, a \rangle \right|^2$$

Translation invariance

$$\langle \Delta x \rangle \equiv \langle x \rangle - x_0 = \nu / N$$



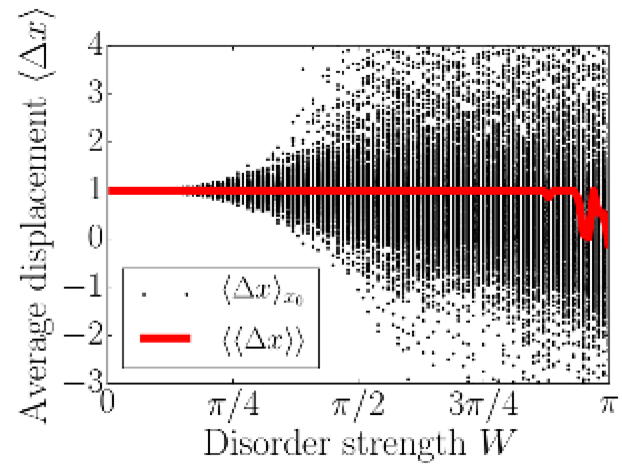


In the disordered case, averaging over initial position is needed: $\langle\langle \Delta x \rangle\rangle = \upsilon$

Disorder → Displacement depends on starting position

So let's average over them!

$$\langle\langle \Delta x \rangle\rangle = \frac{1}{L} \sum_{x_0} \langle \Delta x \rangle_{x_0}$$



Most general statement:

$$\boxed{\langle\langle \Delta x \rangle\rangle = \frac{-2}{LN} \mathrm{Tr} \left\{ \hat{X} \hat{G} \hat{P}_{(E>0)} \right\} = \frac{\nu}{N}}$$

$$\hat{G} = \hat{P}_A - \hat{P}_B$$

We proved ⟨⟨Δx⟩⟩=υ using non-commutative geometry formulation of winding number

Noncommutative geometry for topological insulators: Lori & Hastings, Prodan
for chiral symmetric (AIII): Mondragon-Shem et al, PRL (2014)

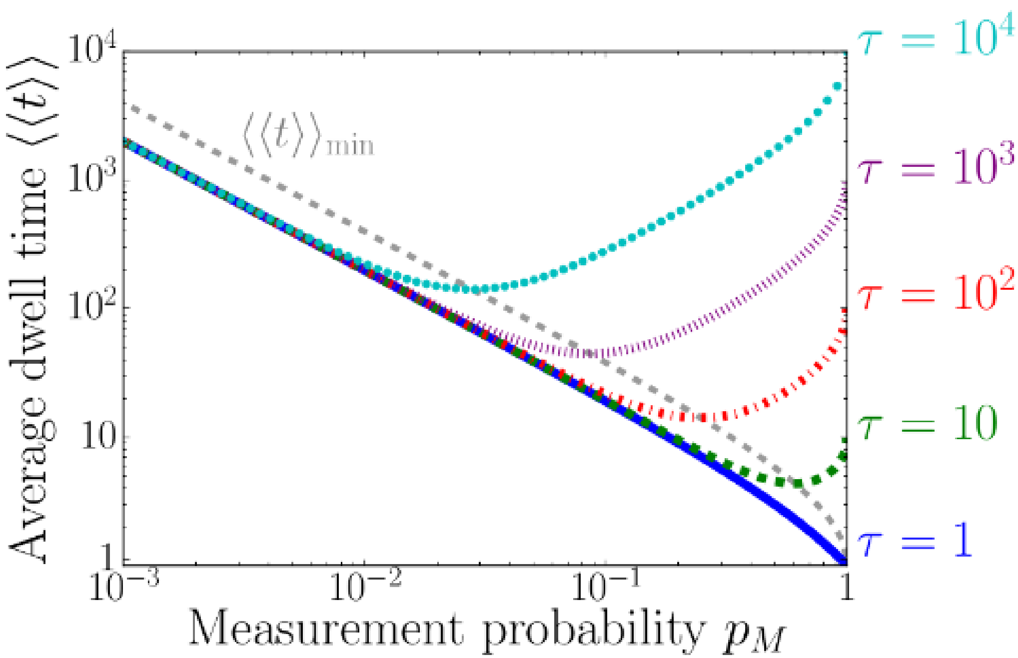
$$\nu = \frac{-(\pi i)^n}{(2n+1)!!} \sum_{\rho} (-1)^{\rho} \mathcal{T} \left\{ \prod_{i=1}^{2n+1} Q_{-+} [X_{\rho_i}, Q_{+-}] \right\}$$

Used this before on quantum walk, compared to scattering formulation of topological invariant [Rakovszky & Asboth, PRA (2015)]

Fast readout can require weak measurement, if almost-dark states are present

Average dwell time:

$$\langle\langle t\rangle\rangle = \frac{p_M}{(1+\sqrt{1-p_M})^2}\underbrace{\int_{E=0}^{\pi}\frac{\rho(E)}{\sin^2 E}\mathrm{d}E}_{\tau} + \frac{2\sqrt{1-p_M}}{p_M}$$



$$\langle\langle t\rangle\rangle|_{p_M=1} = \tau$$

$$\langle\langle t\rangle\rangle_{\min} \approx \sqrt{2\tau} \quad \text{for } \tau \gg 1$$

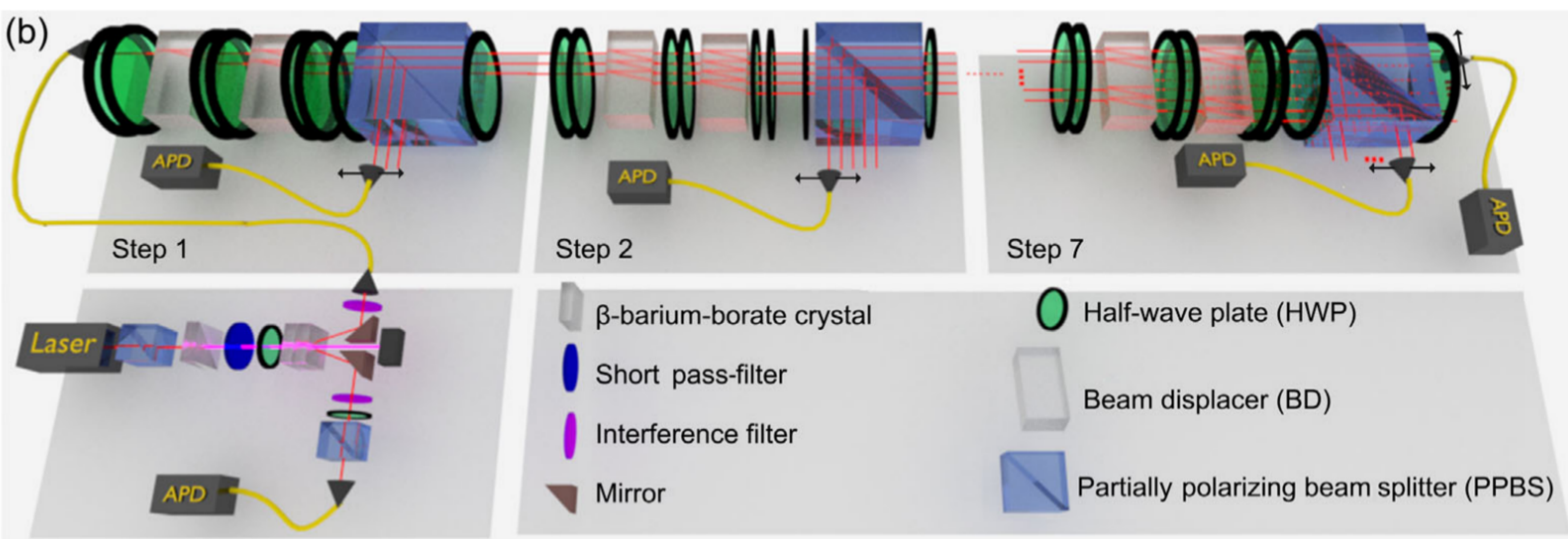
$$p_M^* \approx \sqrt{8/\tau}$$

The experiment we proposed was performed in a quantum walk with single photons

PRL 119, 130501 (2017) PHYSICAL REVIEW LETTERS week ending 29 SEPTEMBER 2017

Detecting Topological Invariants in Nonunitary Discrete-Time Quantum Walks

Xiang Zhan,[1] Lei Xiao,[1] Zhihao Bian,[1] Kunkun Wang,[1] Xingze Qiu,[2,3] Barry C. Sanders,[3,4,5,6] Wei Yi,[2,3,*] and Peng Xue[1,7,†]



Open questions, related work

- Does something like this work in 3 dimensions?
- Massignan & collaborators have since found similar results for $\langle \Delta x \rangle$ defined for Hermitian Hamiltonians, in long-time limit. Precise equivalence?

HUNQUTECH

Acknowledgement for funding: National Research, Development and Innovation Fund of Hungary,
→ FK 124723: *From Topologically Protected States to Topological Quantum Computation*
→ Quantum Technology National Excellence Program (Project Nr. 2017-1.2.1-NKP-2017-00001): *Preparation and distribution of quantum bits, and development of quantum networks*

NATIONAL RESEARCH, DEVELOPMENT AND INNOVATION OFFICE HUNGARY
PROJECT FINANCED FROM THE NRDI FUND
MOMENTUM OF INNOVATION